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Semilinear boundary value problems on a self-adjoint harmonic space with non-local boundary conditions

Dedicated to Professor Yukio Kusunoki on his sixtieth birthday

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Introduction. In the previous paper [2], the author studied semilinear boundary value problems with respect to an ideal boundary on a self-adjoint harmonic space. When applied to the harmonic structure defined by a self-adjoint elliptic operator L on a bounded domain Ω in $\boldsymbol{R}^n$ with smooth boundary $\partial\Omega$, our problem in [2] may be written as

$$(0.1)\qquad \begin{cases} Lu(x) = F(x, u(x)) & \text{on } \Omega, \\ u(\xi) = \tau(\xi) & \text{on } \partial\Omega\setminus\Lambda, \\ \dfrac{\partial u}{\partial n}(\xi) = \beta(\xi, u(\xi)) & \text{on } \Lambda, \end{cases}$$

where F is a function on $\Omega\times\boldsymbol{R}$, Λ is a part of $\partial\Omega$, τ is a given function on $\partial\Omega$ and β is a function on $\Lambda\times\boldsymbol{R}$. The main existence theorem was proved by the so-called monotone-iteration method.

Recently, S. Zheng [3] applied the same method to the following boundary value problem with non-local boundary condition:

$$(0.2)\qquad \begin{cases} Lu(x) = F(x, u(x)) & \text{on } \Omega, \\ u(\xi) = \text{const. (unknown)} & \text{on } \partial\Omega, \\ \displaystyle\int_{\partial\Omega} \frac{\partial u}{\partial n} d\sigma = 0. \end{cases}$$

The purpose of the present paper is to formulate a boundary value problem with respect to an ideal boundary on a self-adjoint harmonic space in such a way that both problems of type (0.1) and of type (0.2) are included as special cases and that the monotone-iteration method can be applied to obtain an existence theorem. In order to describe boundary conditions, we introduce the notion of "boundary behavior spaces". A choice of boundary behavior space gives a problem of the following type, which is a generalization of (0.2):