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Oscillatory properties of the solutions of hyperbolic differential equations with "maximum"

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1. Introduction

Recently there has been a growing interest towards qualitative research of partial differential equations with deviating arguments. However, two papers only have been published so far considering the oscillatory properties of the solutions of partial differential equations with deviating arguments. These are the contributions of D. Georgiou and K. Kreith [1] and of M. Tramov [2].

The present paper studies the oscillatory properties of the solutions of various classes of hyperbolic differential equations with "maximum". Note that the problems for ordinary differential equations with "maximum" find application in the theory for automatic control of various real systems [3], [4]. A. D. Mishkis also points out the necessity to study differential equations with "maximum" in his survey [5]. Oscillatory and asymptotic properties of a class of functional-differential equations with "maximum" have been investigated in the paper of A. Zahariev and D. Bainov [6]. Theorems for existence and uniqueness of the solution of ordinary differential equations with "maximum" have been obtained in [7], [8].

Note that the author is not aware of papers considering partial differential equations with "maximum".

2. Problem of Goursat

In this section we consider the oscillatory properties of the solutions of the problem of Goursat concerning hyperbolic differential equations with "maximum" of the form

$$u_{xy} + p(x, y) \max_{\theta_1 \in [0,\sigma],\, \theta_2 \in [0,\tau]} u(x-\theta_1, y-\theta_2) = 0 \tag{1}$$

where σ, $\tau = const > 0$. Consider the following problem:

To find a solution of equation (1) in the domain $\Pi = \{(x, y)\colon x > 0, y > 0\}$, satisfying the conditions

$$\begin{cases} u(x, y) = \varphi(x, y) & \text{for } (x, y) \in [-\sigma, \infty) \times [-\tau, 0], \\ u(x, y) = \psi(x, y) & \text{for } (x, y) \in [-\sigma, 0] \times [-\tau, \infty). \end{cases} \tag{2}$$