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Standing wave solutions for a Fisher type equation with a nonlocal convection

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Abstract. This paper is concerned with stationary solitary wave solutions of a nonlinear diffusion equation described by

$$u_t = du_{xx} - [(K*u)u]_x + ku(1-u).$$

It is proved that there is no such solution for kernels $K(x) \in L_1(\boldsymbol{R})$, and that for some specific kernel $K(x) \notin L_1(\boldsymbol{R})$ there is a range of value of the total distribution $\int_{\boldsymbol{R}} udx$ for fixed d and k over which such solutions exist.

1. Introduction

Recently there has been considerable interest in biological models governed by a class of reaction-diffusion equations. The simplest model describing the dynamics of one species that moves by diffusion, in one dimension, is expressed by

$$u_t = du_{xx} + k(1-u/\alpha)u, \tag{1}$$

in which the Pearl-Verhulst law is employed for the population growth. Here d is the diffusion constant, k, α are respectively an intrinsic growth rate, a carrying capacity for the species. The equation, which is called the Fisher equation, is also encountered in population genetics. It was analyzed precisely by seeking traveling wave solutions which take the form $u(x, t) = u(x-ct)$ with the constant speed c. As boundary conditions, we have $u(-\infty) = \alpha$, $u(+\infty) = 0$. Traveling wave solutions exist for any fixed $c \geqq c^* = 2\sqrt{kd}$, and a solution of the initial value problem for (1) with a fairly wide class of initial functions forms asymptotically a traveling wave with the minimum speed $c = c^*$. Furthermore, if the initial function is of compact support, it spreads out with time where the fronts move in both directions with the asymptotic speed c^* (see Uchiyama [4]).

On the other hand, we often meet diffusion-convection equations in the absence of growth and/or death terms

$$u_t = du_{xx} - (Vu)_x,$$

where V is the convection velocity of the species. Recently Nagai and Mimura [3] have studied a nonlinear diffusion equation with a nonlocal convection of the form