Hiroshima Math. J.
16 (1986), 1–20

On self H-equivalences of an H-space with respect to any multiplication

To the memory of Shichirô Oka

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(Received January 17, 1985)

Introduction

Let X be an H-space. Then a homotopy equivalence $h: X \to X$ is called a self H-equivalence of X with respect to a multiplication $m: X \times X \to X$ if $hm \sim m(h \times h): X \times X \to X$ (homotopic); and all the homotopy classes of such self H-equivalences form the group

$$\mathrm{HE}(X, m) \text{ (the notation } \mathscr{E}_H(X, m) \text{ is used in the recent papers)}$$

under the composition. In general, X has several multiplications and this group depends on m. For example, the complex conjugate $C: SU(n) \to SU(n)$ of the special unitary group is an H-map with respect to the usual multiplication, but not so to some one on $SU(n)$ for $n \geqq 3$, as is proved by Maruyama-Oka [9].

In this note, we consider the group

$$\mathrm{HE}(X) = \cap_m \mathrm{HE}(X, m) \text{ (} m \text{ ranges over all multiplications on } X\text{)}$$

formed by all self H-equivalences of X with respect to any multiplication, and study its basic properties. The main result is stated as follows:

Theorem. *Let X be the unitary group $U(n)$ $(n \geqq 3)$, the special unitary group $SU(n)$ $(n \geqq 1)$ or the symplectic group $Sp(n)$ $(n \geqq 1)$. Then, any self H-equivalence $h \in \mathrm{HE}(X)$ with respect to any multiplication induces the identity map $h_* = \mathrm{id}$ on $\pi_*(X) \otimes Z_{(p)}$ for a large prime p; and $\mathrm{HE}(X)$ is a finite nilpotent group.*

We prove the basic equality on $\mathrm{HE}(X)$ in Proposition 1.4, and study it in case that X is a product H-space in Theorem 2.4. Furthermore, by using the fact that the localization $X_{(p)}$ of $X = SU(n)$ or $Sp(n)$ at a large prime p is homotopy equivalent to the product space of the localizations of some odd spheres, we study $\mathrm{HE}(X_{(p)})$ in Corollary 3.4; and the main result is proved in Theorem 4.1 and Corollary 4.2 by a similar method to that used in [9].