HIROSHIMA MATH. J.
16 (1986), 405–412

A certain series of unitarizable representations of Lie superalgebras $\mathfrak{gl}(p|q)$

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(Received September 11, 1985)

0. Introduction

In this note, we construct a series of irreducible and unitarizable representations of Lie superalgebras $\mathfrak{gl}(p|q)$ $(1 \leqq p, q \leqq \infty)$. Our representations have two faces — a generalization of discrete series representations for $\mathfrak{su}(n, 1)$ and a supersymmetric analogue of the basic representation of the infinite rank Lie algebra $\mathfrak{gl}(\infty)$.

It is well known that the vertex representation of $\mathfrak{gl}(\infty)$ has deep connections to some areas in non-linear differential equations and mathematical physics such as KP-hierarchies, soliton theory, dual resonance models and statistical models. In section 4, we shall give a Boson picture of our representations for $\mathfrak{gl}(\infty|q)$ $(1 \leqq q \leqq \infty)$, which is a natural extension of the vertex representation of $\mathfrak{gl}(\infty)$.

1. Lie superalgebra $\mathfrak{gl}(p|q)$

Let $\boldsymbol{Z}$ be the set of all integers, and $\boldsymbol{N}$ (resp. $\boldsymbol{N}_0$) the set of all positive (resp. non-negative) integers. We set $\bar{\boldsymbol{N}} = \boldsymbol{N} \cup \{+\infty, \infty\}$, which is a totally ordered set with the natural linear order in $\boldsymbol{N}$ and $n < +\infty < \infty$ for every $n \in \boldsymbol{N}$. For $n \in \bar{\boldsymbol{N}}$, define subsets $\mathscr{S}_n$, $\mathscr{S}_n^*$ and $\mathscr{S}_n^{\pm}$ of $\boldsymbol{Z}$ as follows:

$$\mathscr{S}_n = \begin{cases} \boldsymbol{Z} & \text{if} \quad n = \infty \\ \{j \in \boldsymbol{N}_0 ; j < n\} & \text{if} \quad n < \infty, \end{cases}$$

$$\mathscr{S}_n^* = \mathscr{S}_n - \{0\}, \quad \mathscr{S}_n^{\pm} = \mathscr{S}_n \cap (\pm \boldsymbol{N}).$$

Fix p and q in $\bar{\boldsymbol{N}}$, and define the complex vector spaces $\mathfrak{gl}(p, q)_0$ and $\mathfrak{gl}(p, q)_1$ as follows:

$$\mathfrak{gl}(p, q)_0 = \left\{ \sum_{i,j \in \mathscr{S}_p} a_{ij} E_{ij}^{(00)} + \sum_{k,l \in \mathscr{S}_q} b_{kl} E_{kl}^{(11)} \right\}$$

$$\mathfrak{gl}(p, q)_1 = \left\{ \sum_{(m,n) \in \mathscr{S}_p \times \mathscr{S}_q} c_{mn} E_{mn}^{(01)} + \sum_{(r,s) \in \mathscr{S}_q \times \mathscr{S}_p} d_{rs} E_{rs}^{(10)} \right\},$$

where a_{ij}, b_{kl}, c_{mn} and d_{rs} are complex numbers such that for any integers u and v the number of non-zero a_{ij}, b_{kl}, c_{mn}, d_{rs} with $i, k, m, r > u$ and $j, l, n, s < v$ is finite.