HIROSHIMA MATH. J.
16 (1986), 353–360

Ideal boundary limit of discrete Dirichlet functions

Dedicated to Professor Yukio Kusunoki on his 60th birthday

Maretsugu YAMASAKI

(Received August 20, 1985)

§ 1. Introduction

In the previous paper [5], we proved that every Dirichlet potential $u(x)$ of order $p>1$ on an infinite network $N=\{X, Y, K, r\}$ has limit 0 as x tends to the ideal boundary of N along p-almost every infinite path. Our aim of this paper is to prove the converse of this fact. In case $p=2$, our result has a continuous counterpart in [3], i.e., on a Riemannian manifold Ω, every Dirichlet function (=Tonelli function with finite Dirichlet integral) $u(x)$ has limit 0 as x tends to the ideal boundary of Ω along 2-almost every curve joining a fixed parametric ball to the ideal boundary of Ω if and only if u is a Dirichlet potential (i.e., the values of u on the harmonic boundary of Ω are 0). Since the proof in [3] is based on some results concerning continuous harmonic flows and the Royden compactification of Ω, it seems to be difficult to follow the reasoning in our case.

We shall prove in §2 that every Dirichlet function of order p on X can be decomposed uniquely into the sum of Dirichlet potential of order p and a p-harmonic function on X. We shall discuss in §3 the ideal boundary limit of a non-constant p-harmonic function with finite Dirichlet integral of order p. As an application, we shall prove that a Dirichlet function of order p is a Dirichlet potential of order p if and only if it has limit 0 as x tends to the ideal boundary of N along p-almost every infinite path.

We shall freely use the notation in [5] except for the reference numbers; references are rearranged in the present paper.

§ 2. Decomposition of $D^{(p)}(N)$

Let p and q be positive numbers such that $1/p+1/q=1$ and $1<p<\infty$ and let $\phi_p(t)$ be the real function on the real line R defined by $\phi_p(t)=|t|^{p-1}\,\mathrm{sign}\,(t)$. For each $w\in L(Y)$, let us define $\phi_p(w)\in L(Y)$ by $\phi_p(w)(y)=\phi_p(w(y))$ for $y\in Y$.

For each $u\in L(X)$, the p-Laplacian $\Delta_p u\in L(X)$ of u is defined by

$$\Delta_p u(x) = \sum_{y\in Y} K(x, y)\phi_p(du(y)),$$

where $du(y)=-r(y)^{-1}\sum_{x\in X} K(x, y)u(x)$ (a discrete derivative of u). We say that u is p-harmonic on a subset A of X if $\Delta_p u(x)=0$ on A. Denote by $\boldsymbol{HD}^{(p)}(N)$