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On the asymptotic solutions for a weakly coupled elliptic boundary value problem with a small parameter

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1. Introduction

In this paper we consider the asymptotic solutions of the Dirichlet boundary value problem for a pair of second-order partial differential equations

$$(1.1) \qquad \begin{cases} \varepsilon^2 \Delta u - f(u, v) = 0 \\ \Delta v - g(u, v) = 0 \end{cases}, \qquad x \in \Omega,$$

with the boundary conditions

$$(1.2) \qquad u = \alpha(x, \varepsilon), \quad v = \beta(x, \varepsilon), \quad x \in \Gamma.$$

Here Ω is a bounded domain in $\mathbf{R}^N$ ($N \geq 2$), its boundary Γ is a connected C^∞ hypersurface, and ε is a small positive parameter. Moreover we assume that f and g are infinitely differentiable functions defined in $\mathbf{R}^2$. Such problems arise in the study of steady-state solutions of chemically reacting and diffusing systems (Aris [2] and the references therein). An interesting phenomenon in such systems is the occurrence of boundary and/or interior transition layer. For $N=1$, by using singular perturbation technique, Fife [7] showed that the existence of boundary and/or interior transition layer phenomena for (1.1) and (1.2), and recently Ito [9] modified his results into a more useful version. For $N \geq 2$, as far as we know, these phenomena have not yet been analyzed except numerical simulations. In this paper, we restrict our attention to the problem (1.1) and (1.2) with boundary layer phenomena. Consider the case where the reduced problem

$$\begin{cases} f(u_0, v_0) = 0 \\ \Delta v_0 - g(u_0, v_0) = 0 \end{cases}, \qquad x \in \Omega,$$

and

$$v_0 = \beta_0(x), \quad x \in \Gamma,$$

has a regular solution (u_0, v_0). In this case, we shall show constructively the existence of a solution (u, v) of (1.1) and (1.2) such that

$$\begin{cases} u \to u_0 \text{ uniformly in any compact subset of } \Omega, \end{cases}$$