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Oscillation criteria for functional differential inequalities with strongly bounded forcing term

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1. Introduction

In the last years there has been an increasing interest in studying the oscillatory behaviour of the solutions of differential equations and inequalities which involve forcing terms of the kind introduced by Kartsatos [9, 10]. As examples, we refer the reader to the papers of Chen and Yeh [2–4], Foster [5], Grace and Lalli [6–8], Kartsatos [11], Kusano et al. [12–14], and True [17]. The purpose of this paper is to establish some new oscillation criteria for higher order functional differential inequalities involving more general forcing functions. More precisely, we consider the class of perturbations which represent the so called strongly bounded functions (see [15]).

The functional differential inequalities under consideration are of the form

$$x(t)\{L_n x(t) + f(t, x(g_1(t)),\dots, x(g_m(t))) - h(t)\} \leqq 0,\ n \text{ even}, \tag{1}$$

and

$$x(t)\{L_n x(t) - f(t, x(g_1(t)),\dots, x(g_m(t))) - h(t)\} \geqq 0,\ n \text{ odd}, \tag{2}$$

where $n \geqq 2$ and L_n is the general disconjugate differential operator defined recursively by $L_0 x(t) = a_0(t)x(t)$ and

$$L_k x(t) = a_k(t)(L_{k-1}x(t))', \quad k = 1, 2,\dots, n.$$

We shall assume that $a_i(t)$, $i=0, 1,\dots, n$, are positive and continuous functions on $[t_0, \infty)$ and the operator L_n is in the first canonical form in the sense that

$$\int_{t_0}^{\infty} a_i^{-1}(t)dt = \infty, \quad i = 1, 2,\dots, n-1. \tag{3}$$

In what follows, the set of all real-valued functions $y(t)$ defined on $[t_y, \infty)$ and such that $L_i y(t)$, $i=0, 1,\dots, n$, exist and are continuous on $[t_y, \infty)$ will be denoted by $\mathscr{D}(L_n)$.

For the inequalities (1) and (2) the following conditions will be assumed without further mention: