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On real continuous kernels satisfying the semi-complete maximum principle

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§1. Introduction

According to the so-called Hunt theory, the complete maximum principle is an essential property for a continuous kernel V on a locally compact space X to possess a resolvent and further to be represented by a sub-markovian continuous semi-group $(T_t)_{t>0}$, that is, $Vf=\int_0^\infty T_t f dt$ for any $f \in C_K(X)$ (see, for example, [2] and [13]). While the logarithmic kernel on the 2-dimensional Euclidean space R^2 does not have this property, it satisfies the "*semi-complete maximum principle*" with respect to the Lebesgue measure ξ_2 (see [4]). Furthermore the logarithmic kernel possesses a resolvent and is represented by the 2-dimensional Gauss semi-group in the following sense:

$$\int_{R^2} \log |x-y| f(y) d\xi_2(y) = \int_0^\infty \int_{R^2} \frac{1}{4\pi t} \exp\left(-\frac{|x-y|^2}{4t}\right) f(y) d\xi_2(y) dt$$

for any $f \in C_K(R^2)$ with $\int f d\xi_2 = 0$. Recently, generalizing the logarithmic kernel, M. Itô [4]–[7] considered a real convolution kernel N of logarithmic type on a locally compact abelian group G. By definition, N is "*of logarithmic type*" if there exists a markovian convolution semi-group $(\alpha_t)_{t>0}$ such that $N*f=\int_0^\infty \alpha_t * f dt$ for any $f \in C_K(G)$ with $\int f d\xi = 0$, where ξ is a Haar measure on G. He showed in [4, Théorème A] that a real convolution kernel N is of logarithmic type if and only if

(L.0) N satisfies the semi-complete maximum principle with respect to ξ,

(L.1) $\inf_{x \in G} N*f(x) \leqq 0$ for any $f \in C_K(G)$ with $\int f d\xi = 0$,

(L.2) N is non-periodic,

(L.3) $\lim_{n\to\infty} \eta_{N,CK_n} = -\infty$, where $(K_n)_{n=1}^\infty$ is an exhaustion of G and η_{N,CK_n} is the N-reduced measure of N on CK_n.

In this paper, taking the above fact into consideration, we investigate a real continuous kernel V on a locally compact space X satisfying the semi-complete

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