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Stationary measures for an exclusion process on one-dimensional lattices with infinitely many hopping sites

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§0. Introduction

Since F. Spitzer introduced interacting Markovian particle systems in [11], Markov processes on the configuration space $S^{\mathbf{Z}^d}$ or $S^{\mathbf{R}^d}$ ($S=\{0, 1,\dots, n\}$ or $\{0, 1,\cdots\}$) have been investigated by many authors, and various results are obtained (see, for example, [4, 8, 9]). Those results are, in many cases, about the processes such that the time parameters are continuous and the number of sites in the configuration at which changes occur at the same time is finite. In this paper we consider a Markov process on $\mathscr{X}=\{0, 1\}^{\mathbf{Z}}$ such that the time parameter is discrete and the sites at which changes occur at each time are infinitely many. If we consider the important roles which discrete time stochastic processes play in the theory of probability, it seems worthwhile to investigate discrete time Markov processes in the field of interacting infinite particle systems ([2, 3]).

Let $x\equiv(\cdots x_{-1}x_0x_1\cdots)$ be an element of $\mathscr{X}$. According as $x_i=1$ or 0 we consider that the site i is occupied by a particle or not. Then $x\in\mathscr{X}$ represents a configuration of particles on $\mathbf{Z}$. We introduce a time evolution on $\mathscr{X}$ as follows (for details see §1). Suppose the configuration on $\mathbf{Z}$ at time t is $x\equiv(\cdots x_{-1}x_0x_1\cdots)$. Then as time increases from t to $t+1$ each particle of x moves to the left by one step with probability α $(0<\alpha<1)$ independently when its left site is unoccupied, that is, a particle at site i can move to the site $i-1$ only if $x_{i-1}=0$, and this transition of particle occurs independently in the configuration x. Therefore infinitely many particles can move simultaneously when $\#\{l\colon x_lx_{l+1}=01\}=\infty$. Getting a new configuration $x'\equiv(\cdots x'_{-1}x'_0x'_1\cdots)\in\mathscr{X}$ from x at time $t+1$, we then apply the same transition rule to x' and so on. Thus a time evolution is obtained as a Markov process on $\{0, 1\}^{\mathbf{Z}}$, which we call, following [6, 11], an exclusion process on $\mathbf{Z}$. It should be remarked here that our exclusion process can be thought of as a simple model of semiconductor which is in a (static) electric field if we regard $x_i=1$ and 0 as plus and minus charges respectively.

We define in §1 the transition probabilities of the Markov process stated above precisely and give a sufficient condition (Su) for a translation (=shift) invariant probability measure on $\mathscr{X}$ to be a stationary measure for the process