On strongly increasing entire solutions of even order semilinear elliptic equations

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Introduction

This paper is concerned with entire solutions of even order semilinear elliptic equations of the form

(A) $$\Delta^m u = f(|x|, u, \Delta u, \ldots, \Delta^{m-1} u), \quad x \in R^n,$$

where $m \geq 1$, $n \geq 2$, Δ denotes the n-dimensional Laplacian, $|x|$ is the Euclidean length of x, and f is a given nonnegative continuous function defined on $[0, \infty)^{m+1}$ or on $[0, \infty) \times R^m$. By an entire solution of (A) we mean a function $u \in C^{2m}(R^n)$ which satisfies (A) pointwise in R^n. Important special cases of (A) are

(B) $$\Delta^m u = p(|x|) u^\gamma, \quad x \in R^n,$$

(C) $$\Delta^m u = p(|x|) e^u, \quad x \in R^n,$$

where $\gamma > 1$ and $p: [0, \infty) \to (0, \infty)$ is continuous.

The problem of existence and nonexistence of entire solutions of (A) in the case $m = 1$ has been the subject of intensive investigations in the past three decades, and numerous results have been obtained. Among a vast literature on the subject, we refer the reader to the recent papers [2–5, 10–12, 14–19, 21, 23, 24] which are concerned mainly with second order equations of the forms (B) and (C).

It seems to the author, however, that very little is known about entire solutions for the higher order case of (A) ($m \geq 2$). As far as the author is aware, Walter [26, 27] and Walter and Rhee [28] were the only references in this area until the appearance of Kusano, Naito and Swanson [7–9] and Kusano and Swanson [13], in which a systematic study of the existence and asymptotic behavior of radial entire solutions of (A) with $m \geq 2$ has been attempted. In particular it is shown in [9] that equation (A) ($m \geq 2$) may have a variety of radial entire solutions with different types of asymptotic behavior as $|x| \to \infty$.

The main objective of this paper is to further the theory developed in [7–9] by establishing the existence of a new class of entire solutions for (A). More specifically, we give conditions under which (A) has a radial entire solution $u(x)$ with the asymptotic property