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Removable sets for solutions of the Euler equations

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Introduction

Ahlfors and Beurling [1] gave a characterization of the removable singularities for the class of analytic functions with finite Dirichlet integral, in terms of extremal distances on the complex plane. This result was generalized and extended to the d-dimensional euclidean space R^d ($d \geqq 3$) by many authors (see [3], [6], etc.).

Hedberg [3] gave some characterizations of removable sets for the class HD^p of all harmonic functions u with $\int |\nabla u|^p dx < \infty$ and for the subclass FD^p of HD^p consisting of functions with no flux. In [6] the author considered the notion of null sets for extremal distances of order p, namely, NED_p-sets, and characterized such null sets by the removability for a class of solutions of the Euler equation for the variational integral $\int |\nabla u|^p dx$.

In this paper we shall consider some classes consisting of solutions of the Euler equation for the variational integral $\int \psi(x, \nabla u)dx$, where $\psi(x, \tau)$: $R^d \times R^d \to R$ is strictly convex and continuously differentiable in τ and $\psi(x, \tau) \approx |\tau|^p$, and define the removable sets for these classes. More precisely, for any bounded domain G containing a compact set E, we shall consider the class $\mathscr{H}\mathscr{D}_\psi^p(G-E)$ (resp. $\mathscr{K}\mathscr{D}_\psi^p(G-E)$; $\widetilde{\mathscr{H}\mathscr{D}}_\psi^p(G-E)$) of all p-precise functions u (for p-precise functions, see [4, Chapter IV], [8]) such that

$$\int_{G-E} \langle \nabla_\tau \psi(x, \nabla u), \nabla \phi \rangle dx = 0$$

for every ϕ in $C_0^\infty(G-E)$ (resp. in $\{\phi \in C_0^\infty(G);\ \nabla\phi = 0$ on some neighborhood of $E\}$; in $C_0^\infty(G)$). A compact set E is said to be removable for $\mathscr{H}\mathscr{D}_\psi^p$ (resp. $\mathscr{K}\mathscr{D}_\psi^p$; $\widetilde{\mathscr{H}\mathscr{D}}_\psi^p$) if for some bounded domain G containing E every function in $\mathscr{H}\mathscr{D}_\psi^p(G-E)$ (resp. $\mathscr{K}\mathscr{D}_\psi^p(G-E)$; $\widetilde{\mathscr{H}\mathscr{D}}_\psi^p(G-E)$) can be extended to a function in $\mathscr{H}\mathscr{D}_\psi^p(G)$.

We shall see that E is removable for $\widetilde{\mathscr{H}\mathscr{D}}_\psi^p$ if and only if E is an NED_p-set (Theorem 1). This result is an improvement of [6, Theorem 2]. We shall show that E is removable for $\mathscr{H}\mathscr{D}_\psi^p$ if and only if E is removable for $HD^{p/(p-1)}$ (Theorem 2) and that E is removable for $\mathscr{K}\mathscr{D}_\psi^p$ if and only if E is removable for $FD^{p/(p-1)}$ in case $p \geqq 2$ (Theorem 3). The proofs of these theorems are based on the results obtained by Hedberg [3]. In the case that $\psi(x, \tau) = |\tau|^p$ for all