HIROSHIMA MATH. J.
17 (1987), 79–89

An operator theoretic method for solving $u_t = \Delta\psi(u)$

Yoshikazu KOBAYASHI

(Received May 13, 1986)

1. Introduction

In this paper, we present a new method for solving the Cauchy problem

$$(1.1)\qquad \begin{aligned} &u_t(t, x) = \Delta\psi(u(t, x)), \quad t > 0 \quad \text{and} \quad x \in \boldsymbol{R}^N, \\ &u(0, x) = u_0(x), \ x \in \boldsymbol{R}^N, \end{aligned}$$

where ψ is a locally Lipschitz continuous and nondecreasing function on $\boldsymbol{R}$ such that $\psi(0)=0$; and the method is described from the point of view of the nonlinear semigroup theory.

For $u_0 \in L^1(\boldsymbol{R}^N) \cap L^\infty(\boldsymbol{R}^N)$, a function $u \in L^\infty((0, \infty) \times \boldsymbol{R}^N)$ is called a weak solution of the problem (1.1) if $u \in C([0, \infty); L^1(\boldsymbol{R}^N))$ as an $L^1(\boldsymbol{R}^N)$-valued function on $[0, \infty)$,

$$\int_0^\infty \left(\int_{\boldsymbol{R}^N} u(t, x)f_t(t, x) + \psi(u(t, x))\Delta f(t, x)dx\right)dt = 0$$

for any $f \in C_0^\infty((0, \infty) \times \boldsymbol{R}^N)$ and $u(0, x) = u_0(x)$ a.e.. The existence of weak solutions is established in [1] (in a more general situation) and the uniqueness is proved in [3]. (See also [2] and [11].)

To state the new method for solving the Cauchy problem, let ρ be an arbitrary but fixed rapidly decreasing function on $\boldsymbol{R}^N$ which satisfies

$$(1.2)\qquad \begin{cases} \rho \geq 0, \quad \displaystyle\int_{\boldsymbol{R}^N} \rho(\xi)d\xi = 1, \quad \int_{\boldsymbol{R}^N} \xi_i\rho(\xi)d\xi = 0 \\ \text{and} \\ \displaystyle\int_{\boldsymbol{R}^N} \xi_i\xi_j\rho(\xi)d\xi = \delta_{ij}, \qquad \text{for} \quad i, j = 1, 2,\cdots, N, \end{cases}$$

where $\delta_{ij}=1$ if $i=j$ and $\delta_{ij}=0$ otherwise. (For example, we can choose the (normalized) Gaussian kernel $(2\pi)^{-N/2}\exp(-|\xi|^2/2)$ as such $\rho(\xi)$.) We set

$$(1.3)\qquad \rho_h(\xi, \eta) = \left(\frac{h}{2\psi'_h(\eta)}\right)^{N/2} \rho\left(\left(\frac{h}{2\psi'_h(\eta)}\right)^{1/2}\xi\right)$$

for $(\xi, \eta) \in \boldsymbol{R}^N \times \boldsymbol{R}$ and $h > 0$, where $\{\psi_h\}_{h>0}$ is a family of smooth strictly increasing functions on $\boldsymbol{R}$ such that $\psi_h(0)=0$, $\psi_h(\eta)\to\psi(\eta)$ as $h \downarrow 0$, uniformly for bounded