HIROSHIMA MATH. J.
17 (1987), 67–77

A Hausdorff-Young inequality for the Fourier transform on Riemannian symmetric spaces

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(Received May 13, 1986)

§ 1. Introduction

Let G/K be a Riemannian symmetric space of non-compact type. In [1] spherical Fourier transforms of left K-invariant L^p $(1<p<2)$ functions on G/K are studied and it is shown that the spherical transforms of these functions are extended holomorphically to a certain domain T_p, which is determined only by p, in $\mathfrak{a}_C^*$ and a Hausdorff-Yong inequality holds. We adopt $\pi_\nu(f)=\int_G f(x)\pi_\nu(x)dx$ as the Fourier transform of $f\in C_0^\infty(G/K)$; here π_ν denotes the induced representation of class one from the minimal parabolic subgroup P of G. The purpose of this paper is to show that the Fourier transforms of K-finite L^p functions on G/K also satisfy a Hausdorff-Young type inequality in the domain T_p similar to the spherical case.

§ 2. Notation and Preliminaries

Let G be a connected semisimple Lie group with finite center and $\mathfrak{g}$ its Lie algebra. We denote by $\langle\cdot,\cdot\rangle$ the Killing form of $\mathfrak{g}$. Let $G=KAN$ be an Iwasawa decomposition and $\mathfrak{k}$, $\mathfrak{a}$ and $\mathfrak{n}$ the Lie subalgebras of $\mathfrak{g}$ corresponding to K, A and N respectively. Each $x\in G$ can be written uniquely as $x=\kappa(x)\cdot \exp H(x)n(x)$, where $\kappa(x)\in K$, $H(x)\in\mathfrak{a}$ and $n(x)\in N$. Let M' and M be the normalizer and the centralizer of $\mathfrak{a}$ in K respectively and denote by $W=M'/M$ the Weyl group. Throughout this paper, we denote the dual space of a real or complex vector space V by V^* and the complexification of a real vector space V by V_C. We fix an ordering on $\mathfrak{a}^*$ which is compatible with the above Iwasawa decomposition. Let Σ denote the set of all positive roots of $(\mathfrak{g}, \mathfrak{a})$ and $m(\alpha)$ the multiplicity of $\alpha\in\Sigma$. Let Σ_0 be the set of elements in Σ which are not integral multiples of other elements in Σ. We put $a(\alpha)=m(\alpha)+m(2\alpha)$ for $\alpha\in\Sigma_0$ and $\rho=2^{-1}\sum_{\alpha\in\Sigma}m(\alpha)\alpha$. Let $\mathfrak{a}_+^*$ be the positive Weyl chamber of $\mathfrak{a}^*$ and put

$$\mathfrak{a}_+ = \{H\in\mathfrak{a}\,|\,\alpha(H)>0 \text{ for all } \alpha\in\mathfrak{a}_+^*\};\quad A^+ = \exp\mathfrak{a}_+ .$$

For any $\varepsilon\geq 0$, we put

$$C_{\varepsilon\rho} = \{\lambda\in\mathfrak{a}^*\,|\,|(s\lambda)(H)|\leq\varepsilon\rho(H) \text{ for all } H\in\mathfrak{a}_+ \text{ and } s\in W\}.$$