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Positive entire solutions of semilinear biharmonic equations

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1. Introduction

Our main objective is to prove the existence of infinitely many positive, radially symmetric entire solutions $u(x)$ of the fourth order semilinear elliptic differential equation

$$(1.1) \qquad \Delta^2 u = f(|x|, u), \quad x \in \mathbf{R}^N, \quad N \geqq 3,$$

where Δ denotes the N-dimensional Laplacian, $|x|$ denotes the Euclidean length of x, and f is a real-valued continuous function in $[0, \infty) \times (0, \infty)$. An *entire solution* of (1.1) is defined to be a function $u \in C^4(\mathbf{R}^N)$ satisfying (1.1) pointwise in $\mathbf{R}^N$. Detailed hypotheses on (1.1) to be used for existence theorems of three different types are listed in §2.

Emphasis will be placed on the prototype

$$(1.2) \qquad \Delta^2 u = p(|x|)u^\gamma, \quad x \in \mathbf{R}^N,$$

where $\gamma \neq 1$ is a real constant and $p: [0, \infty) \to \mathbf{R}$ is a continuous function satisfying one of the following three decay conditions:

$$(1.3) \qquad \int^\infty t^{2\gamma+1}|p(t)|dt < \infty, \quad N \geqq 3;$$

$$(1.4) \qquad \int^\infty t^3|p(t)|dt < \infty, \quad N \geqq 5;$$

$$(1.5) \qquad \int^\infty t^\delta p(t)dt < \infty, \quad N \geqq 5,$$

where $\delta = N - 1 - \gamma(N-4)$, $-1 < \gamma < 1$, and $p(t) \geqq 0$ in (1.5). Our results establish, in particular, the existence of infinitely many positive entire solutions of (1.2) of *each* of the following three types under conditions (1.3), (1.4), or (1.5), respectively:

(I) Unbounded entire solutions $u(x)$ which are bounded above and below by positive constant multiples of $1 + |x|^2$;

(II) Entire solutions which are bounded above and below by positive constants; and

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