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Block one-step methods for starting multistep methods

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1. Introduction

Consider the initial value problem

$$y' = f(x, y), \quad y(x_0) = y_0, \tag{1.1}$$

where the function $f(x, y)$ is assumed to be sufficiently smooth. Let $y(x)$ be the solution of this problem, let

$$x_t = x_0 + th \qquad (t>0,\ h>0) \tag{1.2}$$

and denote by y_t an approximation to $y(x_t)$, where h is a stepsize. Multistep methods for solving (1.1) numerically require starting values. For instance, a linear k-step method needs y_t $(t=1, 2,\dots, k-1)$ (see [2]), and a two-step method with one off-step node x_v $(0<v<1)$ requires y_v and y_1 (see [6]).

Rosser [3] and Shintani [4, 5] have proposed block one-step methods of the form

$$y_t = y_0 + h \textstyle\sum_{i=1}^{q} p_{it} k_i \tag{1.3}$$

that provide y_t for $t=1, 2,\dots, N$, where

$$k_1 = f(x_0, y_0), \tag{1.4}$$

$$k_i = f(x_0 + a_i h,\ y_0 + h \textstyle\sum_{j=1}^{i-1} b_{ij} k_j) \qquad (i=2, 3,\dots, q), \tag{1.5}$$

$$\textstyle\sum_{j=1}^{i-1} b_{ij} = a_i, \quad a_i \neq 0 \qquad (i=2, 3,\dots, q), \tag{1.6}$$

p_{kt} $(k=1, 2,\dots, q)$, a_i and b_{ij} $(j=1, 2,\dots, i-1;\ i=2, 3,\dots, q)$ are constants. Rosser has shown that for $q=N(N+3)/2$ there exists a method (1.3) of order $N+1$ for $t=1, 2,\dots, N$ and that for $t=N=2p$ the order can be raised to $N+2$ with one more evaluation of f. Shintani [5] has proved that for $q=4$, 6 there exists a method (1.3) which is of order 3, 4 for $t=1$ and is of order 4, 5 for $t=2$ respectively. But these methods cannot be used to start multistep methods with off-step nodes. Gear [1] has considered methods of the form (1.3) that can provide y_t for any t's and has shown that there exists a method (1.3) of order 3 for $q=4$ but none for $q=3$, that a method of order 4 exists for $q=6$ and that q must not be less than 9 for constructing a method of order 5.