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Free boundary problems for some reaction-diffusion equations

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§ 1. Introduction

The present paper deals with a free boundary problem which models regional partition phenomena arising in population biology. Our problem is to look for a family of functions $\{u(x, t), s(t)\}$ $((x, t) \in [0, 1] \times [0, \infty))$ which satisfy

$$u_t = d_1 u_{xx} + uf(u) \quad \text{in} \quad S^-, \tag{1.1}$$

$$u_t = d_2 u_{xx} + ug(u) \quad \text{in} \quad S^+, \tag{1.2}$$

$$u(0, t) = 0 \quad \text{for} \quad t \in (0, \infty), \tag{1.3}$$

$$u(1, t) = 0 \quad \text{for} \quad t \in (0, \infty), \tag{1.4}$$

$$u(s(t), t) = 0 \quad \text{for} \quad t \in (0, \infty), \tag{1.5}$$

$$\dot{s}(t) = -\mu_1 u_x(s(t)-0, t) + \mu_2 u_x(s(t)+0, t) \quad \text{for} \quad t \in (0, \infty) \quad \text{where} \quad 0 < s(t) < 1, \tag{1.6}$$

$$u(x, 0) = \varphi(x) \quad \text{for} \quad x \in I \equiv (0, 1), \tag{1.7}$$

$$s(0) = l, \tag{1.8}$$

where $x = s(t)$ corresponds to a free boundary, S^- (resp. S^+) is an open subset of $I \times (0, \infty)$ in which $x < s(t)$ (resp. $x > s(t)$), d_i and $\mu_i (i = 1, 2)$ are positive constants, $\dot{s}(t)$ denotes $(d/dt)s(t)$ and $u_x(s(t)-0, t)$ (resp. $u_x(s(t)+0, t)$) means the limit of $u(x, t)$ at $x = s(t)$ from the left (resp. right). For the derivation of the free boundary problem (1.1)–(1.8), we refer the reader to [8].

In (1.1) and (1.2), f and g are assumed to possess the following properties:

(A.1) f is locally Lipschitz continuous on $[0, \infty)$ and satisfies $f(1) = 0$ and $f(u) \leqq 0$ on $[1, \infty)$.

(A.2) g is locally Lipschitz continuous on $(-\infty, 0]$ and satisfies $g(1) = 0$ and $g(u) \leqq 0$ on $(-\infty, -1]$.

On the initial data $\{\varphi, l\}$ we put the following conditions:

(A.3) $0 \leqq l \leqq 1$.

(A.4) $\varphi \in H_0^1(I)$ satisfies $\varphi(l) = 0$ and $(l - x)\varphi(x) \geqq 0$ for $x \in \bar{I} = [0, 1]$.