HIROSHIMA MATH. J.
17 (1987), 521–533

Lie algebras in which every 1-dimensional subideal is an ideal

Hidekazu FURUTA and Takanori SAKAMOTO

(Received January 17, 1987)

Introduction

A Lie algebra L is called a t-algebra if every subideal of L is an ideal of L, a T-algebra if any subalgebra of L is a t-algebra and a c-algebra if every nilpotent subideal of L is an ideal of L. We easily see that L is a c-algebra if and only if every 1-dimensional subideal of L is an ideal of L. Recently Varea [14] introduced the concept of C-algebra in Lie algebra: L is a C-algebra if every subalgebra of a nilpotent subalgebra H of L is an ideal in the idealizer of H in L. He investigated the property of finite-dimensional C-algebras, and in [14] he proved the following results:

(a) Let L be an n-dimensional Lie algebra over a field $\mathfrak{k}$ of at least $n-1$ elements. Then the following are equivalent: i) L is a C-algebra. ii) L is a T-algebra. iii) Every subalgebra of L is a c-algebra.

(b) Let L be a finite-dimensional Lie algebra over a field of characteristic zero. Then the following are equivalent: i) L is a c-algebra. ii) L is a t-algebra. iii) $L=R\oplus S$ where R is an ideal of L which is either abelian or almost-abelian and S is a semisimple ideal of L.

The purpose of this paper is to give several generalizations of (a), (b) and other results in [14] without the finite-dimensionality of L and the restriction on the cardinality of $\mathfrak{k}$.

The main results of this paper are as follows.

(1) Let L be a serially finite Lie algebra over a field of characteristic zero. If the locally soluble radical of L belongs to the class $\acute{E}(\mathrm{si})\mathfrak{A}$ of Lie algebras, then the three statements in (b) are equivalent (Theorem 2.3).

(2) Let L be an arbitrary Lie algebra. Then the following are equivalent: i) L is a C-algebra. ii) Every subalgebra of L is a c-algebra. iii) Every 1-dimensional ascendant subalgebra of a subalgebra H of L is an ideal of H (Theorem 3.5).

(3) Let L be a locally finite Lie algebra over any field. Then the following are equivalent: i) L is a C-algebra. ii) L is a T-algebra. iii) Every serial subalgebra of a subalgebra H of L is an ideal of H. iv) Every 1-dimensional serial subalgebra of a subalgebra H of L is an ideal of H (Theorem 3.9).

(4) Over any field there exist a c-algebra which is neither a C-algebra nor a