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The first eigenvalue of the Laplacian for the compact quotient of a certain Riemannian symmetric space

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§ 1. Introduction

Let (M, g) be a Riemmanian symmetric space of noncompact type. Then M is isometric to a coset space G/K where G is a non-compact semisimple Lie group with finite center and K is a maximal compact subgroup of G. Put $o = \{K\} \in M$. Normalize g in such a way that it is induced by the Killing form of the Lie algebra of G. Let Γ be a discrete subgroup of G acting fixed point freely on M whose quotient manifold $M_\Gamma = \Gamma \backslash M$ is compact. Let Δ_Γ be the Laplacian (cf. [1]) acting on C^∞ functions on M_Γ for the Riemannian metric induced by g. The compactness of M_Γ implies that the spectrum of Δ_Γ forms a discrete subset of the set of non-negative real numbers. Let $\lambda_1(\Gamma)$ denote the the first positive eigenvalue of Δ_Γ. Consider all such cocompact discrete subgroups Γ of G. Then we know the following inequality for several (M, g)'s,

$$\limsup_{\mathrm{vol}(M_\Gamma)\to\infty} \lambda_1(\Gamma) \leq |\rho|^2,$$

where the positive constant $|\rho|^2$ depends only on M (cf. §2). When (M, g) is the unit disc with the Poincaré metric, H. Huber showed this inequality in [6]. H. Urakawa in [7] generalized it to the case when (M, g) is a Riemannian symmetric space of noncompact type of rank one.

The purpose of the present article is to show this inequality when (M, g) is the Riemannian symmetric space of noncompact type such that G is complex.

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§ 2. Preliminaries

Let $M = G/K$ be a Riemannian symmetric space of noncompact type where G is a semisimple Lie group with finite center and K is a maximal compact subgroup of G. Let $\mathfrak{g}$ and $\mathfrak{k}$ denote the Lie algebras of G and K respectively. Let B denote the Killing form of $\mathfrak{g}$ and let $\mathfrak{p}$ denote the orthogonal complement of $\mathfrak{k}$ in $\mathfrak{g}$ with respect to B. Then $\mathfrak{g} = \mathfrak{k} + \mathfrak{p}$ is the Cartan decomposition and $\mathfrak{p}$ is identified with the tangent space T_0M. We assume that the Riemannian metric g on M is