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The Gangolli estimates for the coefficients of the Harish-Chandra expansions of the Eisenstein integrals on real reductive Lie groups

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1. Introduction

1.1 SUMMARY

Let G and $\mathfrak{g}$ be a Lie group and its Lie algebra. We denote by $\mathfrak{g}_c$ and G_c the complexification of $\mathfrak{g}$ and the complex adjoint group respectively. In this paper we assume that G is of class $\mathscr{H}$, that is, G satisfies the following three conditions: (1) $\mathfrak{g}$ is reductive and $Ad(G)\subset G_c$; (2) the center of the analytic subgroup corresponding to $[\mathfrak{g}, \mathfrak{g}]$ is finite; (3) the number of connected components of G is finite.

As is well known, the Eisenstein integrals on G, that is, the matrix elements of representations of principal series for G, play an essential role in harmonic analysis on G. Therefore it is very important to know the asymptotic behaviors of the Eisenstein integrals. In fact, the leading terms of the expansions of these integrals give the Harish-Chandra C-functions and are closely related to the Plancherel measure (cf. [5], [6]). The analysis of the Schwartz space on G needs only the leading terms as an approximation of the Eisenstein integrals and estimates of difference between them (cf. [1], [2], [6]). On the other hand, to carry out closer study of harmonic analysis on G, such as Paley-Wiener type theorems for various function spaces, one needs to know the asymptotic behavior of higher order, of the Eisenstein integrals and to estimate the approximations (cf. [2], [7], [8], [9], [10]). If we use our results to prove the Paley-Wiener type theorem for the L^p Schwartz spaces, as showed in [3], we can get it without the approximation theorems such as in [2], [8] or [10]. For an application, see Eguchi and Wakayama [4].

For each $\nu\in\mathscr{F}_C$, the zonal spherical function is defined by

$$\varphi_\nu(x) = \int_K e^{(\nu-\rho)(H(xk))}dk \quad (x\in G).$$

(The notation will be explained later.) When $x=h$ varies in the positive Weyl chamber A^+ of A, $\varphi_\nu(h)$ is expanded into an infinite series (cf. Harish-Chandra [5]) as