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On the boundary limits of harmonic functions

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1. Introduction

This paper deals with the boundary behavior of harmonic functions u on a bounded open set $G \subset R^n$ satisfying

$$\int_G |\text{grad}\, u(x)|^p \omega(x)dx < \infty,$$

where $p > 1$ and ω is a nonnegative measurable function on G. The function $\omega(x)$ is mainly of the form $\varphi(d(x))$, where $d(x)$ denotes the distance of x from the boundary ∂G and φ is a monotone function on the interval $(0, \infty)$. Moreover, G is assumed to satisfy certain smoothness conditions mentioned later.

Our first aim in this paper is to find a positive function $A(x)$ on G for which $A(x)u(x)$ tends to zero as x tends to the boundary ∂G. We shall next give conditions which assure the boundedness of u on G or near a boundary point of G. In special cases, u will be shown to have a finite limit at a boundary point; our discussion below will include the proof of the existence of nontangential limits.

We here remark that the case $p=1$ can be treated similarly with a small modification.

2. Boundary limits of harmonic functions on general bounded domains

Throughout this paper, let G be a bounded domain in R^n satisfying the following condition: There exist a compact set K and a positive number c such that any point x in G is joined to K by a piecewise smooth curve $x(t)$ in G having the following properties:

(C_1) $x(1) \in K$. (C_2) $x(0) = x$.

(C_3) $|x(t_2) - x(t_1)| \leqq c(t_2 - t_1)|x(0) - x(1)|$ whenever $0 \leqq t_1 \leqq t_2 \leqq 1$.

(C_4) $|x(t_2) - x(t_1)| \geqq c^{-1}(t_2 - t_1)|x(0) - x(1)|$ whenever $0 \leqq t_1 \leqq t_2 \leqq 1$.

(C_5) If $y \in B(x(t), 2^{-1}d(x(t)))$, then $d(x) + |x - y| < cd(y)$.

REMARK. Condition (C_4) implies the following:

(C_6) For any $y \in G$, the linear measure of the set of all t such that $y \in B(x(t), 2^{-1}d(x(t)))$ is dominated by $M|x(0) - x(1)|^{-1}d(y)$,