Almost complex and almost contact structures in fibred Riemannian spaces

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Introduction

The idea of foliations of manifolds emerged from works of C. Ehresmann and G. Reeb in the 1940's. One of directions of its study in differential geometry and topology has been to research possibility of fibering of manifolds, and many works in this field were synthetically reviewed by H. B. Lawson Jr. [5] and B. L. Reinhart [13].

Another direction is to research structures and properties of fibred space, and it goes back to a unified field theory in a 5-dimensional Riemannian space due to Th. Kaluza and O. Klein. Fibred Riemannian spaces were first considered by Y. Muto [7] and treated by B. L. Reinhart [12] in the name of foliated Riemannian manifolds. B. O'Neill [11] called such a foliation a Riemannian submersion and gave its structure equations and in the almost same time K. Yano and S. Ishihara [20, 21, 22] developed an extensive theory of fibred Riemannian spaces. M. Ako [1] and T. Okubo [10] studied fibred spaces with almost complex or almost Hermitian structure. These works were synthetically reported in S. Ishihara and M. Konishi's monograph [4].

In connection with almost contact structure, S. Tanno [17] and Y. Ogawa [9] investigated principal bundles over almost complex spaces having a 1-dimensonal structure group. Generalizing Calabi-Eckmann's example, S. Morimoto [6] defined an almost complex structure in the product of two almost contact spaces and obtained a condition on the normality, and S. Goldberg and K. Yano [3] researched similar properties for the product of two framed manifolds.

On the other hand, the tangent bundle of a Riemannian space can be endowed with a Riemannian metric, see S. Sasaki [14, 15], and with an almost complex structure associated with the Riemannian connection, see S. Tachibana and M. Okumura [16]. P. Dombroski [2], T. Nagano [8], S. Tanno [18], Y. Tashiro [19], I. Yokote [23] and other geometers investigated properties of tangent bundles related with metric and almost complex structure.

In this point of view, the purpose of the present paper is to study relations among structures of fibred spaces with almost Hermitian or almost contact metric structures and those of base spaces and fibres, and to apply results to tangent bundles of Riemannian spaces. In §§1–3, we shall explain preliminaries