HIROSHIMA MATH. J.
18 (1988), 95–111

On Hasse-Witt matrices of Fermat varieties

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(Received May 19, 1987)

Introduction

Let X be an n-dimensional Fermat variety of degree d

$$x_0^d + x_1^d + \cdots + x_{n+1}^d = 0 \quad (d \geqq n+2)$$

in $\mathbf{P}^{n+1}$, where $x_0, x_1, \dots, x_{n+1}$ are homogeneous coordinates. We are concerned with the p-th power frobenius action F on the n-th cohomology group $H^n(X, \mathcal{O}_X)$ of X over an algebraic closure k of the field $\mathbf{F}_p$ ($p>0$; $p \nmid d$). The F-module $H^n(X, \mathcal{O}_X)$ is canonically isomorphic to the G_h-module $H^{n+1}(\mathbf{P}^{n+1}, \mathcal{O}_{\mathbf{P}^{n+1}}(-d))$, and we know that the vector space $H^{n+1}(\mathbf{P}^{n+1}, \mathcal{O}_{\mathbf{P}^{n+1}}(-d))$ has as basis $\mathscr{W}_0$ (cf. §1). We now consider the matrix (the so-called Hasse-Witt matrix) $\mathrm{HW}(X)$ of G_h with respect to $\mathscr{W}_0$.

In this paper, we show mainly the following theorems:

THEOREM I. *For positive integers n, d and p (p; prime number with $p \nmid d$ and $d \geqq n+2$) given as above, we let ρ_i be the number of all elements in $\mathscr{W}_0$ of type i defined in* §1. *We can arrange the ρ_i's by some integers $f_0 > f_1 > \cdots > f_r > 0$ as follows*:

$$\rho_i = 0 \quad \text{for} \quad i > f_0, \quad \rho_{f_s} = \rho_i < \rho_{f_{s+1}} \quad \text{for} \quad f_s \geqq i > f_{s+1}$$
$$\text{and} \quad s < r, \quad \rho_{f_r} = \rho_i \leqq \rho_0 \quad \text{for} \quad f_r \geqq i \geqq 1.$$

We denote by $\mathrm{HW}(X)_{nilp}$ the nilpotent part of $\mathrm{HW}(X)$ at p. Then the normal form of $\mathrm{HW}(X)_{nilp}$ becomes the matrix

$$\begin{pmatrix} \Lambda(1) & & & & & & 0 \\ & \Lambda(2) & & & & & \\ & & \ddots & & & & \\ & & & \Lambda(\rho_{f_r}) & & & \\ & & & & 0 & & \\ & & & & & 0 & \\ 0 & & & & & & \ddots \\ & & & & & & & 0 \end{pmatrix} \left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} \rho_0 - \rho_{f_r}$$

with $\Lambda(\rho) = \Lambda_{f_\alpha + 1}$ for $\rho_{f_{\alpha-1}} < \rho \leqq \rho_{f_\alpha}$, $\alpha = 0, 1, \dots, r$, where $\rho_{f_{-1}} = 0$, and each