On Hasse-Witt matrices of Fermat varieties

Keisuke Toki

(Received May 19, 1987)

Introduction

Let $X$ be an $n$-dimensional Fermat variety of degree $d$

$$x_0^d + x_1^d + \cdots + x_n^d = 0 \quad (d \geq n + 2)$$

in $\mathbb{P}^{n+1}$, where $x_0, x_1, \ldots, x_{n+1}$ are homogeneous coordinates. We are concerned with the $p$-th power Frobenius action $F$ on the $n$-th cohomology group $H^n(X, \mathcal{O}_X)$ of $X$ over an algebraic closure $k$ of the field $\mathbb{F}_p$ ($p > 0; \ p \nmid d$). The $F$-module $H^n(X, \mathcal{O}_X)$ is canonically isomorphic to the $G_{\Lambda}$-module $H^{n+1}(\mathbb{P}^{n+1}, \mathcal{O}_{\mathbb{P}^{n+1}}(-d))$, and we know that the vector space $H^{n+1}(\mathbb{P}^{n+1}, \mathcal{O}_{\mathbb{P}^{n+1}}(-d))$ has as basis $\mathcal{W}_0$ (cf. §1). We now consider the matrix (the so-called Hasse-Witt matrix) $HW(X)$ of $G_{\Lambda}$ with respect to $\mathcal{W}_0$.

In this paper, we show mainly the following theorems:

THEOREM I. For positive integers $n$, $d$ and $p$ ($p$; prime number with $p \nmid d$ and $d \geq n + 2$) given as above, we let $\rho_i$ be the number of all elements in $\mathcal{W}_0$ of type $i$ defined in §1. We can arrange the $\rho_i$'s by some integers $f_0 > f_1 > \cdots > f_s > 0$ as follows:

$$\rho_i = 0 \quad \text{for} \quad i > f_0, \quad \rho_{f_s} = \rho_i < \rho_{f_{s+1}} \quad \text{for} \quad f_s \geq i > f_{s+1}$$

and $s < r, \quad \rho_{f_r} = \rho_i \leq \rho_0 \quad \text{for} \quad f_r \geq i \geq 1$.

We denote by $HW(X)_{\text{nilp}}$ the nilpotent part of $HW(X)$ at $p$. Then the normal form of $HW(X)_{\text{nilp}}$ becomes the matrix

$$\begin{pmatrix}
\Lambda(1) & 0 \\
\Lambda(2) & \vdots \\
& \Lambda(\rho_{f_r}) \\
& 0 \\
0 & 0 & \cdots & 0 \\
\end{pmatrix}_{\rho_0 - \rho_{f_r}}$$

with $\Lambda(\rho) = \Lambda_{\rho_{f_{\alpha+1}}}$ for $\rho_{f_{\alpha+1}} < \rho \leq \rho_{f_{\alpha}}$, $\alpha = 0, 1, \ldots, r$, where $\rho_{f_{-1}} = 0$, and each