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Boundary behavior of p-precise functions on a half space of R^n

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1. Introduction

Let u be a function which is locally p-precise in $D=\{x=(x_1,\dots, x_n);\ x_n>0\}$, $n\geqq 2$, and satisfies

$$(1)\qquad \int_D |\mathrm{grad}\, u(x)|^p x_n^\alpha dx < \infty, \quad 1 < p < \infty, \quad -1<\alpha<p-1$$

(see Ohtsuka [12] for (locally) p-precise functions). Many authors have tried to find a set $F\subset D$ such that $u(x)$ has a finite limit as x tends to the boundary ∂D along F (see Aikawa [1], Carleson [2], Mizuta [5], [7], [8], [9], Wallin [13]). They were mainly concerned with the nontangential case, that is, the case where $F=\ell_\xi\equiv\{\xi+(0, t);\ t>0\}$ or $F=\Gamma(\xi, a)\equiv\{x=(x', x_n)\in R^{n-1}\times R^1;\ |x'-\xi'|<ax_n\}$; if $u(x)$ has a finite limit as $x_n \downarrow 0$ along ℓ_ξ, then u is said to have a perpendicular limit at ξ, and if $u(x)$ has a finite limit as $x\to\xi$ along $\Gamma(\xi, a)$ for any $a>0$, then u is said to have a nontangential limit at ξ. The existence of tangential limits of u at ξ was discussed by Aikawa [1] and Mizuta [9]. The proof of the existence of these limits can be carried out by local arguments; in fact it requires to find conditions near ξ which assure the existence of limits.

In this paper we investigate a global behavior of u near the boundary ∂D. More precisely, we aim to find a function $A(x)$ such that $A(x)u(x)$ tends to zero as x tends to ∂D along a set $F\subset D$. In order to evaluate the size of F, we use the capacity:

$$C_p(E;\ G) = \inf \|f\|_p^p,$$

where the infimum is taken over all nonnegative measurable functions f on R^n such that $f=0$ outside G and $\int_G |x-y|^{1-n}f(y)dy\geqq 1$ for every $x\in E$; $\|\cdot\|_p$ denotes the L^p-norm in R^n. As in Aikawa [1], we introduce a notion of thinness of a set in D, near the boundary ∂D; we say that a set E is *C_p-thin near ∂D* if there exists a positive integer j_0 such that

$$\text{in case } p < n, \quad \textstyle\sum_{j=j_0}^\infty 2^{j(n-p)}C_p(E_j;\ D) < \infty,$$

$$\text{in case } p = n, \quad \textstyle\sum_{j=j_0}^\infty C_p(E_j\cap G_1;\ G_2) < \infty$$