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Dirichlet integral and energy of potentials on harmonic spaces with adjoint structure

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Introduction

On a self-adjoint harmonic space, we can establish Green's formulae, which give relations between Dirichlet integral and energy of potentials (see [6; Part II]). On the other hand, for the potential theory with respect to the heat equation, relations between Dirichlet integral and energy have not been completely clarified; on a cylindrical domain, such relations have been discussed by M. Pierre [7], [8] in the framework of "parabolic Dirichlet space", and also some results can be found in the investigations of parabolic capacities (see, e.g., [4]).

The purpose of the present paper is to obtain such relations on a P-harmonic space having an adjoint structure. We shall establish a sort of Green's formula for continuous potentials with finite energy and show that the Dirichlet integral is majorized by the energy for such potentials.

In the last section, we shall investigate the case of the heat equation on a cylindrical domain $X=\Omega\times(0, T)$ ($\Omega\subset \boldsymbol{R}^d$, $d\geq 1$) and show that continuous heat potentials with finite energy on X belong to the class $L^2(0, T; H_0^1(\Omega))\cap L^\infty(0, T; L^2(\Omega))$, which is a space considered in [7], [8] (also, cf. [4]).

§1. Mutually adjoint harmonic structures

Let X be a connected locally compact space with a countable base, and suppose two harmonic sheaves $\mathscr{H}$ and $\mathscr{H}^*$ (or hyperharmonic sheaves $\mathscr{U}$ and $\mathscr{U}^*$) are given so that $(X, \mathscr{H})$ and $(X, \mathscr{H}^*)$ (or, $(X, \mathscr{U})$ and $(X, \mathscr{U}^*)$) are both P-harmonic spaces in the sense of Constantinescu-Cornea [2]. The set of all continuous potentials with respect to $\mathscr{H}$ (resp. $\mathscr{H}^*$) is denoted by $\mathscr{P}_C$ (resp. $\mathscr{P}_C^*$). We say that $\mathscr{H}$ and $\mathscr{H}^*$ are *mutually adjoint* if there exists a function (called an associated Green function) $G(x, y)\colon X\times X\to[0, +\infty]$ satisfying the following conditions:

(G.0) $G(x, y)$ is lower semicontinuous on $X\times X$ and continuous off the diagonal set;

(G.1) For each $y\in X$, $G(\cdot, y)$ is a potential for $\mathscr{H}$ and is harmonic for $\mathscr{H}$ on $X\setminus\{y\}$;