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# Numerical interfaces in nonlinear diffusion equations with finite extinction phenomena

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## 1. Introduction

From a numerical point of view, we study the following nonlinear degenerate diffusion equation

$$v_t = (v^m)_{xx} - cv^p, \quad (t, x) \in (0, \infty) \times \mathbf{R}^1 \tag{1.1}$$

with an initial condition

$$v(0, x) = v^0(x), \quad x \in \mathbf{R}^1, \tag{1.2}$$

where $m$ $(>1)$, $c$ $(>0)$ and $p$ $(>0)$ are all constants and $v^0$ is a nonnegative continuous function with a compact support.

Equations of the form (1.1) are known as models in the fields of fluid dynamics [13], plasma physics [1] and population dynamics [5]. For instance, (1.1) describes a one-dimensional nonlinear fluid-transfer process with an absorption, where $v = v(t, x)$ is the density of fluid at time $t$ and place $x$. The most interesting phenomenon, which (1.1), (1.2) exhibits, is the occurrence of the finite propagation of the initial support. As is well known, in the absence of $-cv^p$, the support of the solution $S(t) \equiv \operatorname{supp} v(t, \cdot)$ expands and finally becomes unbounded as time increases. On the other hand, in the presence of $-cv^p$, which implies volumetric absorption, it is already shown that the behavior of $S(t)$ (in other words, *interfaces*) is qualitatively classified into the following three cases depending on $m$ and $p$ (see [2], [6], [7], [8], [9] and [10]):

(1) For $p \geqq m$, $S(t)$ expands as $t$ increases and

$$S(t) \longrightarrow \mathbf{R}^1, \quad \text{as} \quad t \longrightarrow \infty.$$

(2) For $1 \leqq p < m$, $S(t)$ also expands and there exists a bounded set $\mathrm{B} \subset \mathbf{R}^1$ satisfying

$$S(t) \subset \mathrm{B}, \quad \text{for all} \quad t \geqq 0.$$

(3) For $0 < p < 1$, $S(t)$ is compact in $\mathbf{R}^1$ and there exists a positive number $T^* < \infty$ such that