HIROSHIMA MATH. J.
18 (1988), 361–372

Unbounded nonoscillatory solutions of nonlinear ordinary differential equations of arbitrary order

Takaŝi KUSANO and Manabu NAITO

(Received September 3, 1987)

1. Introduction

Consider the differential equation

$$y^{(n)} + \sigma f(t, y, y', \dots, y^{(n-1)}) = 0, \tag{1.1}$$

where $n \geqq 2$, $\sigma = +1$ or -1, and f: $[0, \infty) \times \boldsymbol{R}^n \to \boldsymbol{R}$ is a continuous function such that

$$y_0 f(t, y_0, y_1, \dots, y_{n-1}) \geqq 0 \qquad \text{for} \quad (t, y_0, y_1, \dots, y_{n-1}) \in [0, \infty) \times \boldsymbol{R}^n. \tag{1.2}$$

Let $\mathscr{N}$ denote the set of all nonoscillatory solutions of (1.1), that is, those solutions which are defined in some neighborhood of infinity and are eventually positive or negative. We denote by $\mathscr{N}_k$, $0 \leqq k \leqq n$, the set of all $y \in \mathscr{N}$ satisfying the inequalities

$$(1.3)_k \qquad \begin{cases} y(t)y^{(i)}(t) > 0, & t \geqq T_y, \quad 0 \leqq i \leqq k-1, \\ (-1)^{i-k} y(t) y^{(i)}(t) \geqq 0, & t \geqq T_y, \quad k \leqq i \leqq n \end{cases}$$

for $T_y > 0$ sufficiently large. Such an $\mathscr{N}_k$ is often referred to as a Kiguradze class for (1.1). Of basic importance is the fact [4, 5] that, under condition (1.2), every nonoscillatory solution $y \in \mathscr{N}$ of (1.1) falls into one and only one Kiguradze class $\mathscr{N}_k$ with k such that

$$n \not\equiv k \pmod 2 \quad \text{if} \quad \sigma = +1, \quad \text{and} \quad n \equiv k \pmod 2 \quad \text{if} \quad \sigma = -1; \tag{1.4}$$

in other words, $\mathscr{N}$ has the following decomposition:

$$\begin{aligned} &\mathscr{N} = \mathscr{N}_1 \cup \mathscr{N}_3 \cup \cdots \cup \mathscr{N}_{n-1} && \text{for} \quad \sigma = +1 \quad \text{and} \quad n \quad \text{even}, \\ &\mathscr{N} = \mathscr{N}_0 \cup \mathscr{N}_2 \cup \cdots \cup \mathscr{N}_{n-1} && \text{for} \quad \sigma = +1 \quad \text{and} \quad n \quad \text{odd}, \\ &\mathscr{N} = \mathscr{N}_0 \cup \mathscr{N}_2 \cup \cdots \cup \mathscr{N}_n && \text{for} \quad \sigma = -1 \quad \text{and} \quad n \quad \text{even}, \\ &\mathscr{N} = \mathscr{N}_1 \cup \mathscr{N}_3 \cup \cdots \cup \mathscr{N}_n && \text{for} \quad \sigma = -1 \quad \text{and} \quad n \quad \text{odd}. \end{aligned}$$

Note that (1.4) is equivalent to $(-1)^{n-k-1}\sigma = 1$.

The study of Kiguradze classes has been one of the central problems in