HIROSHIMA MATH. J.
18 (1988), 309–339

On the structure of connection coefficients for hypergeometric systems

Toshiaki YOKOYAMA

(Received September 2, 1987)

Introduction

In this paper we shall be concerned with a connection problem for the so-called hypergeometric system of linear differential equations

$$(0.1)\qquad (t-B)\frac{dX}{dt} = AX \quad (t \in \boldsymbol{C}),$$

where X is an n-dimensional column vector, A is an n by n constant matrix and B is an n by n diagonal matrix. This is a Fuchsian system with regular singularities at diagonal elements of B and infinity in the whole complex t-plane.

The global study of (0.1) was initiated by K. Okubo [9], who investigated an effective method of algebraic computation of the monodromy group for (0.1) without accessory parameters, together with the reduction of every single Fuchsian differential equation to (0.1) ([10], see also [7]). R. Schäfke [12] and W. Balser-W. B. Jurkat-D. A. Lutz [1] cleared up the relation between connection coefficients of (0.1) and the Stokes multipliers of the Birkhoff system of linear differential equations

$$z\frac{dY}{dz} = \{-(1+A)+Bz\}Y,$$

which has a regular singularity at $z=0$ and an irregular singularity of rank 1 at $z=\infty$, through the Laplace transformation $Y(z)=\int X(t)e^{zt}dt$.

Recently M. Kohno [5] has shown that the connection problem for (0.1) can be solved by a global analysis of the system of linear difference equations

$$(0.2)\qquad (B-\lambda)(z+1)G(z+1) = (z-A)G(z) \quad (z \in \boldsymbol{C})$$

which gives the coefficients in power series solutions of (0.1). In [6] he has also analyzed a case when there appear logarithmic solutions at finite singularities and has shown the global Frobenius theorem. By means of the method of [5], the author [14] (see also [13]) has analyzed completely (0.1) in the case when A is diagonalizable and has only two distinct eigenvalues, and has verified the following results:

(i) Principal solutions of (0.2) in the right half z-plane give the solutions of