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Derivation of the Boltzmann equation from particle dynamics

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Contents

Introduction

This paper is exclusively concerned with the equation of Boltzmann (or Maxwell and Boltzmann) for a gas of identical hard spheres and its caricature. In his original derivation of the equation Boltzmann made crucial use of an assumption of *molecular chaos* or so-called *stosszahlansatz* which, groundlessly introduced, is acceptable for no better reason than that it is plausible or expedient and, lacking in precision of its meaning, obscures the relations of the Boltzmann equation to the underlying dynamics — while has been recognized its significance in the kinetic theory of gases, the Boltzmann equation hardly rested on any solid foundation. It therefore was (and is) highly desirable to derive the Boltzmann equation from the Liouville equation, i.e., to derive it from particle dynamics which is completely deterministic (causal) so that all the randomness introduced into the derivation comes only through the *initial randomness* of the particle configuration in the phase space. Through arguments (though not of mathematical rigor) based on the careful analysis of the Liouville equation and its reduced equation for the one particle correlation function H. Grad (1958) afforded an excellent insight into the nature of "stosszahlansatz", pointing out, among others, the crutial role played by very small parts of the phase space through which the behavior of a correlation function is determined by that of higher order ones.