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A note on the inequality $\Delta u \geqq k(x)e^u$ in R^n

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1. Introduction

This note is concerned with the problem of nonexistence of entire solutions for the differential inequality

$$(1) \qquad \Delta u \geqq k(x)e^u, \qquad x \in \boldsymbol{R}^n,$$

where $n \geqq 2$, Δ is the n-dimensional Laplacian and $k(x)$ is a nonnegative continuous function in $\boldsymbol{R}^n$. An entire solution $u(x)$ of inequality (1) is defined to be a real-valued function of class $C^2(\boldsymbol{R}^n)$ which satisfies (1) at every point of $\boldsymbol{R}^n$. The following result was established by Oleinik [5]:

THEOREM 0. *Suppose that $k(x) \geqq \theta(|x|)|x|^{-2}$ for large $|x|$, where $|\cdot|$ denotes the Euclidean length, $\theta(t) \to \infty$ as $t \to \infty$ and $\theta(t)t^{-2}$ is a nonincreasing function of t. Then inequality* (1) *has no entire solution.*

The purpose of this note is to improve and extend this result. First, we derive nonexistence criteria for (1), sharper than Oleinik's, on the basis of the consideration of certain ordinary differential inequalities. Then we attempt to obtain an extension of Theorem 0 to more general elliptic inequalities of the form (16). For other related results, we refer the reader to the papers [2, 3, 4, 6] and the references contained therein.

2. Results

First, we introduce the notation

$$k_*(r) = \min_{|x|=r} k(x) \qquad \text{for} \quad r \geqq 0,$$

and for an entire solution $u(x)$ of (1), we put

$$\bar{u}(r) = \frac{1}{\omega_n r^{n-1}} \int_{|x|=r} u(x)dS \qquad \text{for} \quad r \geqq 0,$$

where ω_n denotes the surface area of the unit sphere in $\boldsymbol{R}^n$, i.e., $\bar{u}(r)$ is the spherical mean of $u(x)$ over the sphere $|x| = r$. An improvement of Theorem 0 in the two-