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On Lie algebras in which every subalgebra is a subideal

Masanobu HONDA

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Introduction

Heineken and Mohamed [4] have constructed a Fitting, metabelian group with trivial centre in which every subgroup is subnormal. In Lie theory, Unsin [10] has constructed a Fitting, metabelian Lie algebra with trivial centre in which every subalgebra is a subideal. As in group theory, the class $\mathfrak{D}$ of Lie algebras in which every subalgebra is a subideal is one of the typical classes of generalized nilpotent Lie algebras.

Recently Brookes [2] has proved that a hyperabelian group in which no non-trivial section is perfect and in which every subgroup is subnormal, is soluble ([2, Theorem A]), and he has concluded that a hypercentral group in which every subgroup is subnormal, is soluble ([2, Corollary A]). Subsequently, generalizing [2, Theorem A], Casolo [3] has proved that a group in which no non-trivial section is perfect and in which every subgroup is subnormal, is soluble ([3, Theorem]). The purpose of this paper is to present the Lie-theoretic analogues of [2, Theorem A and Corollary A] and [3, Theorem].

We shall first prove that $\mathfrak{D}\cap\acute{\mathrm{E}}(\lhd)\mathfrak{A}\cap(\grave{\mathrm{E}}\mathfrak{A})^{\mathrm{Q}}\leqq\mathrm{E}\mathfrak{A}$ (Corollary 1), where $\acute{\mathrm{E}}(\lhd)\mathfrak{A}$ is the class of hyperabelian Lie algebras, $(\grave{\mathrm{E}}\mathfrak{A})^{\mathrm{Q}}$ is the largest Q-closed subclass of the class of hypoabelian Lie algebras and $\mathrm{E}\mathfrak{A}$ is the class of soluble Lie algebras. The group-theoretic analogue of this result is also true and is a slight generalization of [2, Theorem A]. We shall secondly prove that over any field $\mathfrak{k}$ of characteristic zero $\mathfrak{D}\cap(\grave{\mathrm{E}}\mathfrak{A})^{\mathrm{QS}}\leqq\mathrm{E}\mathfrak{A}$ (Theorem 2), where $(\grave{\mathrm{E}}\mathfrak{A})^{\mathrm{QS}}$ is the largest Q-, S-closed subclass of the class of hypoabelian Lie algebras and is equal to the class of Lie algebras in which no non-trivial section is perfect.

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Throughout this paper we always consider not necessarily finite-dimensional Lie algebras over a field $\mathfrak{k}$ of arbitrary characteristic unless otherwise specified. Notations and terminology are based on [1]. But for the sake of convenience we list the terms that we use here.

Let L be a Lie algebra over a field $\mathfrak{k}$ and n be a non-negative integer. By $H\leqq L$ (resp. $H\lhd L$, $H \operatorname{ch} L$, $H\lhd^{n}L$, $H \operatorname{si} L$), we mean that H is a subalgebra (resp. an ideal,