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Comparison of powers of a class of tests for covariance matrices

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1. Introduction

This paper is concerned with problems of testing the hypotheses (i) for the equality of covariance matrix to a given matrix, (ii) for the sphericity and (iii) for the equality of two covariance matrices. When the underlying distribution is normal, the commonly used tests for testing these hypotheses are the likelihood ratio (=LR) tests. Anderson [1], Sugiura ([10], [11], [12]) and Nagao ([6], [8]) derived the asymptoic expansions of their null and non-null distributions. Nagao ([7], [9]) proposed certain test statistics for testing the above hypotheses and derived the asymptotic expansions of their null and non-null distributions. Hayakawa [3] proposed a modified Wald statistic for a simple hypothesis when underlying distribution is more general. He made the comparison of some tests for the problem (i) under local alternatives.

Let the $p \times 1$ vectors $X_1, \dots, X_N$ be a random sample from a normal distribution with mean vector μ and covariance matrix Σ. The modified LR criterion for testing the hypothesis $\mathscr{H}: \Sigma = \Sigma_0$ against the alternatives $\mathscr{K}: \Sigma \neq \Sigma_0$ for some given positive definite matrix Σ_0, is given by

$$\lambda = |\Sigma_0^{-1} S|^{n/2} \operatorname{etr}\{-(n/2)(\Sigma_0^{-1} S - I)\}, \tag{1.1}$$

where $S = n^{-1} \sum_{j=1}^{N} (X_j - \bar{X})(\bar{X}_j - \bar{X})'$, $\bar{X} = N^{-1} \sum_{J=1}^{N} X_j$ and $n = N-1$. Wald statistic is given by

$$T_1 = (n/2)\operatorname{tr}(S^{-1}\Sigma_0 - I)^2. \tag{1.2}$$

The test statistic proposed by Nagao [7] is given by

$$T_2 = (n/2)\operatorname{tr}(\Sigma_0^{-1} S - I)^2. \tag{1.3}$$

These three statistics are the symmetric functions of latent roots of $\Sigma_0^{-1} S$. Let $d_1, \dots, d_p$ be the latent roots of $\Sigma_0^{-1} S$. It is seen that $(d_1, \dots, d_p)$ is a maximal invariant under a certain group of transformations (see e.g., Muirhead [5]). We consider a class C of ststistics

$$T = n \sum_{j=1}^{p} Q(d_j), \tag{1.4}$$