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Riesz potentials, higher Riesz transforms and Beppo Levi spaces

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§1. Introduction

Let R^n be the n-dimensional Euclidean space and f be a continuous function on R^n with compact support. For a positive integer l with $2l<n$, a solution of the equation

(1.1) $$\Delta^l u = c_{l,n} f$$

is given by

$$U_{2l}^f(x) = \int |x-y|^{2l-n} f(y)dy,$$

where $c_{l,n} = (2l-n)(2l-2-n)\cdots(2-n)2^l(l-1)!\pi^{n/2}/\Gamma(n/2)$. The function U_m^f is called the Riesz potential of order m of f. In particular, U_2^f is the Newton potential of f. Naturally, the following problem arises: Find a representation of a solution of the equation (1.1) for any positive integer l and any L^p-function f. We note here that for an L^p-function f, U_m^f does not necessarily exist in case $m-(n/p)\geqq 0$.

Let m be a positive integer and $p>1$. We denote by $\mathscr{L}_m^p$ the space of all distributions u such that $D^\alpha u \in L^p$ for any $|\alpha| = m$. If $m-(n/p)<0$, then $u\in\mathscr{L}_m^p$ can be represented as follows ([12]):

(1.2) $$u(x) = \sum_{|\gamma|\leqq m-1} a_\gamma x^\gamma + U_m^f(x), \quad f\in L^p.$$

We are also concerned with the following problem: For any positive integer m and p