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Correlation effects for stochastic zeros of Sturm-Liouville equations

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1. Introduction

Stochastic Sturm-Liouville equations of the form

$$(p(x, \omega)u')' + q(x, \omega)u = 0; \quad (x, \omega) \in [0, \infty) \times \Omega \tag{1.1}$$

arise naturally in mathematical models of vibrating systems whose physical properties (e.g. masses and spring constants) are known only in terms of probabilities. The most obvious deterministic approximation to such an equation is the classical Sturm-Liouville equation

$$(P(x)v')' + Q(x)v = 0; \quad x \in [0, \infty) \tag{1.2}$$

where $P(x)$ and $Q(x)$ are the expected values of $p(x, \omega)$ and $q(x, \omega)$ relative to a given probability space Ω. The rather natural correspondence between (1.1) and (1.2) makes it important to understand the extent to which solutions of (1.2) do in fact approximate solutions of (1.1).

This paper is concerned with solutions of (1.1) and (1.2) which also satisfy initial conditions of the form

$$u(0, \omega) = 0 \quad \text{with probability } 1; \quad v(0) = 0.$$

Denoting the smallest positive zeros of such solutions of (1.1) and (1.2) by $\xi(\omega)$ and η, respectively, we shall focus on the relationship between X and η, where X denotes the expected value of $\xi(\omega)$ relative to Ω.

In [3] an analogous theory for eigenvalues serves as a basis for establishing criteria which assure that

$$X \geqslant \eta. \tag{1.3}$$

While applying to very general classes of equations, the criteria of [3] do not take into account the nature of the correlation between $p(x, \omega)$ and $q(x, \omega)$ and, as a result, lead to rather restrictive hypotheses for assuring (1.3). The present paper focuses on the nature of the correlation between $p(x, \omega)$ and $q(x, \omega)$ in establishing criteria for (1.3).

It is assumed throughout that the coefficients $p(x, \omega)$ and $q(x, \omega)$ are