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On the oscillatory properties of the solutions of non-linear neutral functional differential equations of second order

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1. Introduction

In the present paper sufficient conditions have been obtained for oscillation or tending to zero of all bounded solutions of equations of the form

$$[A(x_t)]'' + p(t)B(x_t) = 0\,, \tag{1}$$

where $x_t(\theta) = x(t+\theta)$, $\theta \in [-\tau, 0]$, $\tau = \text{const} > 0$ and the functionals A, B: $C[-\tau, 0) \to \boldsymbol{R}$ are monotonic.

The oscillatory properties of linear and non-linear ordinary differential and functional differential equations have been an object of investigation by many authors [2]–[5], [8], [10]. The neutral equations of second order have numerous applications (see for instance [1], [6]) but their oscillatory and asymptotic properties are studied comparatively little. Some results in this direction for the case when the function $p(t)$ is nonnegative have been obtained in [9], [11], [12].

2. Preliminary notes and main result

DEFINITION 1. We shall say that the function $\varphi: J_\varphi \to \boldsymbol{R}$ ($J_\varphi = [t_\varphi, \infty)$, $t_\varphi \in \boldsymbol{R}$)) is oscillating if $\sup\{t \mid \varphi(t) = 0\} = \infty$ and $\sup\{t \mid \varphi(t) \neq 0\} = \infty$.

DEFINITION 2. A function $x: J_x \to \boldsymbol{R}$ will be called a solution of equation (1) if $x \in C(J_x)$, $A(x_t) \in C^2(J_x + \tau)$ and satisfies equation (1) for $t \in J_x + \tau$, where $J_x + \tau = \{t \mid t - \tau \in J_x\}$.

By $\Omega^{\alpha,\beta}$ ($0 < \beta \leqq \alpha$) we shall denote the set of all continuous functionals $A: C[-\tau, 0] \to \boldsymbol{R}$ which satisfy the following conditions:

A1. For any function $\varphi \in C[-\tau, 0]$ with the property $\varphi(t) \neq 0$, $t \in [-\tau, 0]$, the following equality holds

$$\operatorname{sgn} A(\varphi) = \operatorname{sgn} \varphi(0)\,.$$