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Abstract quasi-linear equations of evolution in nonreflexive Banach spaces

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Introduction

In [7] T. Kato established the existence of classical solutions to the abstract quasi-linear equations of the form

$$(CP)\qquad \begin{aligned} &du(t)/dt + A(t, u(t))u(t) = f(t, u(t)), \quad 0 \leq t \leq T, \\ &u(0) = a, \end{aligned}$$

and applied his theory to a wide variety of problems from mathematical physics. In his theory two reflexive Banach spaces X and Y are used in such a way that Y is continuously and densely embedded in X, the solution $u(t)$ of (CP) lies in some open subset W of Y and $du(t)/dt$ is found in X. The reflexivity of X (and hence that of Y) is essential for the theory, and it is important from the theoretical point of view to eliminate this restriction for the spaces X and Y. Furthermore, in order to apply the theory to partial differential equations in suitable function spaces, it is required to extend this theory to the case of nonreflexive Banach spaces.

The first purpose of the present paper is to establish an existence theorem for the classical solutions of (CP) in a pair of general Banach spaces $X \supset Y$. We shall show that the solutions are continuous in Y-norm and continuously differentiable in X. In order to construct such solutions, we employ the following type of difference approximation of (CP):

$$(D)\qquad \begin{aligned} &\frac{u_\Delta(t) - u_k}{t - t_k} + A(t_k, u_k)u_\Delta(t) = f(t_k, u_k), \quad t_k \leq t \leq t_{k+1}, \quad 0 \leq k \leq N-1, \\ &u_\Delta(0) = a, \end{aligned}$$

where $\Delta: 0 = t_0 < t_1 < \cdots < t_N = T$ is a partition of the interval $[0, T]$ and $u_k = u_\Delta(t_k)$. This approach is one of the main features of our argument. In case the operators $A(t, w)$ and $f(t, w)$ are independent of t and of the form $A(w)$ and $f(w)$, it was proved in [14] that for any initial value $a \in W$, one finds a $T > 0$ such that the solution u_Δ of (D) exists in W, and that $\{u_\Delta\}$ converges in X to a unique weak solution (in the sense of [13]) of (CP) as $|\Delta| = \max(t_k - t_{k-1}) \to 0$. In this t-independent case the proof of the convergence of $\{u_\Delta(t)\}$