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Relations between several Adams spectral sequences

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Introduction

In the stable homotopy theory, the G-Adams spectral sequence

(1) $E(G) = \{E(G)_r^{s,t}, d_r\colon E(G)_r^{s,t} \to E(G)_r^{s+r,t+r-1}\}$ abutting to $\pi_{t-s}(X)$

(cf. [4, III, §15]) is useful, where X is a CW spectrum, $\pi_*(X)$ is its homotopy group and G is a ring spectrum. For X and $G = E$, F with some conditions, H. R. Miller [10] introduced the May and Mahowald spectral sequences

(2) $E^{\mathrm{May}} = \{E_{u,r}^{s,t}, d_r^{\mathrm{May}}\colon E_{u,r}^{s,t} \to E_{u+r,r}^{s+1,t+r}\}$ abutting to $E(E)_2^{s,u-t}$ and
$E^{\mathrm{Mah}} = \{\tilde{E}_{u,r}^{s,t}, d_r^{\mathrm{Mah}}\colon \tilde{E}_{u,r}^{s,t} \to \tilde{E}_{u,r}^{s+r,t-r+1}\}$ converging to $E(F)_2^{s+t,u}$

for $E(G)_2$ in (1), which satisfy the following

(o) $E_{u,1}^{s,t} = \tilde{E}_{u,2}^{s,t} = A_u^{s,t}$; *and for any* $x \in A_u^{s,t}$,
(ii) *if x converges to x^F in E^{Mah}, then so does $d_1^{\mathrm{May}}x$ to $(-1)^t d_2^F x^F$.*

Especially, he defined these algebraically in case when
(3) $X = S^0$, $E = BP$ at a prime p, and $F = HZ_p$ (BP is the Brown-Peterson spectrum, and HZ_p is the spectrum of the ordinary homology $H_*(\ ;Z_p)$); and calculated some differential $d_2^{HZ_p}$ in (1) for $X = S^0$.

The purpose of this paper is to argue the existence and relations of these spectral sequences. Let $\bar{G}$ denote the mapping cone of the unit $S^0 \to G$ of a ring spectrum G, and $\bar{G}^n$ the smash product of n copies of $\bar{G}$. Then the main result in this paper, stated in Theorem 7.2, implies the following

THEOREM. *For a CW spectrum X and ring spectra E, F, assume that*
(4) *there is a unit-preserving map $\lambda\colon E \to F$, and*
(5) *the F-Adams spectral sequence abutting to $\pi_*(E \wedge \bar{E}^n \wedge X)$ in* (1) *converges and collapses for any $n \geqq 0$.*

Then we have the spectral sequences E^{May} and E^{Mah} in (2) *satisfying* (o), (ii),
(i) $d_1^{\mathrm{May}} d_2^{\mathrm{Mah}} x = d_2^{\mathrm{Mah}} d_1^{\mathrm{May}} x$ *for any* $x \in A_u^{s,t}$,
(iii) *if x converges to x^E in E^{May}, then so does $d_2^{\mathrm{Mah}}x$ to $d_2^E x^E$, and*
(iv) *if the assumptions in* (ii)–(iii) *hold, then some $y \in A_{u+1}^{s+2,t}$ converges to $d_2^E x^E$ in E^{May} and to $(-1)^t d_2^F x^F$ in E^{Mah}.*