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Cyclic Galois extensions of regular local rings

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§1. Introduction

Let R be a formal power series ring in d indeterminates over an algebraically closed field, and let L be a finite, abelian Galois extension of the field K of fractions of R such that the order of the Galois group is prime to the characteristic of K. Let S be the integral closure of R in L. As proved in [2], S is a free R-module of rank $n = |G|$, and hence it is a Cohen-Macaulay local ring of dimension d.

The R-algebra structure of a free R-module S defines structural constants $g(\chi, \chi') \in R$, where χ and χ' run through all characters of G(see §2); our main theorem in this note, Theorem 7 in §4, gives a condition which characterizes the invertibility of $g(\chi, \chi')$'s, and consequently, it gives a method to calculate the embedding dimension and the Cohen-Macaulay type of S. In the case that L is a cyclic Galois extension, we shall make a detailed discussion in §5; more precisely, we can compute these two numerical invariants whenever a defining equation $z^n = f$, $f \in R$, of the extension L over K is given.

Notation and terminology.

For a commutative ring A, A^* will denote the group of invertible elements in A.

Throughout this paper, R will be a noetherian domain containing an algebraically closed field K, L will be a finite Galois extension of the field K of fractions of R. We denote by G the Galois group of L over K. S will be the integral closure of R in L; we say that S is a Galois extension of R. We assume that R is a unique factorization domain (UFD), *G is abelian* and *$n = |G|$ is invertible in k.*

A character of an abelian group means a group homomorphism from it to k^*. Since the Galois group G is abelian, the set $\mathrm{Hom}(G, k^*)$ of all characters of G forms a group which is isomorphic to G; we denote by $\chi_1, \cdots, \chi_n$ the characters of the Galois group G. If H is a finite abelian group such that $(|H|, \mathrm{char}\ k) = 1$, for a character χ of H, we put $e(\chi) = n^{-1}\sum_{\sigma \in H} \chi(\sigma^{-1})\sigma$; $e(\chi)$ is an element in the group ring $k[H]$.