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A predator-prey diffusion model in age-dependent population dynamics

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1. Introduction

In this paper we study an age-dependent predator-prey model with spatial movement in which the predator species has a tendency to eat the prey species in the specific age-interval. Age-dependent diffusion models for a single biological species were first proposed by Gurtin [4], where the population density of the species $u = u(a, t, x)$ is governed by the following:

$$\boldsymbol{D}u + \lambda u = k\Delta u \tag{1.1}$$

with the boundary condition at $a = 0$

$$u(0, t, x) = \int_0^\infty \beta u(a, t, x)\, da\,. \tag{1.2}$$

Here a stands for age, t for time and x for spatial position. The operator $\boldsymbol{D}$ is defined by

$$(\boldsymbol{D}u)(a, t, x) = \lim_{h \to 0} \frac{1}{h}(u(a + h, t + h, x) - u(a, t, x))\,. \tag{1.3}$$

The functions λ and β denote the death modulus and the birth modulus of the population, respectively, both of which in general depend on a, t, x, u itself and the total population density

$$P(t, x) = \int_0^\infty u(a, t, x)\, da\,. \tag{1.4}$$

The equation (1.1) is the so-called balance equation and (1.2) describes the birth process of the species.

Nonlinear age-dependent population models (including several interacting populations) without spatial diffusion have been studied by many authors (see, for instance, Gurtin and MacCamy [5], Webb [11] and the references therein). The existence and uniqueness of solutions in age-dependent diffusion models for a single species has been also investigated (see, for instance, Busenberg and Iannelli [1], Di Blasio [2], Kunisch, Schappacher and Webb [7] and MacCamy