Oscillations of mixed neutral equations

I. P. Stavroulakis

(Received May 16, 1988)

1. Introduction and preliminaries

A first order functional differential equation in which the present value of \(x(t)\) is expressed in terms of both past and future values of \(x\) is said to be of "mixed" type. A first order equation in which the expression for \(\dot{x}(t)\) involves \(\dot{x}(\tau(t))\) for some \(\tau(t) \neq t\) is said to be of "neutral" type. So, when both of these characteristics are present, the equation is of mixed neutral type or a neutral equation with mixed arguments or simply a mixed neutral equation. See Driver [6].

Consider the neutral differential equation

\[
d\frac{d}{dt} [x(t) + cx(t - r)] + \sum_{i=1}^{k} p_{i} x(t - \tau_{i}) = 0
\]

where \(c, r, p_{i}, \tau_{i}, i = 1, \ldots, k\) are real numbers.

Observe that when \(c = 0\) or \(r = 0\) the above equation reduces to a non-neutral equation whose oscillatory character has been studied by several authors. See for example the papers by Ladas and Stavroulakis [22, 23], Arino, Győri and Jawhari [1], Hunt and Yorke [16] and Fukagai and Kusano [7]. Also in the case where \(p_{i} = 0, i = 1, \ldots, k\) equation (*) reduces to

\[
d\frac{d}{dt} [x(t) + cx(t - r)] = 0
\]

and there exists a (nonoscillatory) solution of the form \(x(t) = c, c\) a constant. Thus we will assume that

\[c \neq 0, \quad r \neq 0, \quad \text{and} \quad p_{i} \neq 0 \quad \text{for all} \quad i = 1, \ldots, k.\]

The following (duality) lemma is easily established (cf. [11, Lemma 5]).

**Lemma 1.1.** Suppose that \(c \neq 0\) and \(p_{i} \neq 0, i = 1, \ldots, k\). Then \(x(t)\) satisfies the neutral equation (*) if and only if \(x(t)\) satisfies the neutral equation

\[
d\frac{d}{dt} \left[ x(t) + \frac{1}{c} x(t - (-r)) \right] + \frac{1}{c} \sum_{i=1}^{k} p_{i} x(t - (\tau_{i} - r)) = 0.
\]