HIROSHIMA MATH. J.
20 (1990), 421–424

Maximal tori and the center in an analytic group

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(Received September 9, 1989)

§ 1. Introduction

Let G be an analytic group (= connected Lie group), and Z the center of G. Let G' denote the factor group G/Z, which can be identified with the adjoint group of G. In [1] and [2], the author introduced notions of "*generalized maximal tori*" and "*standard Cartan subgroups*" of G, in terms of the adjoint group of G. They played important roles in these papers. Each of these subgroups is connected with a maximal torus of the adjoint group, and contains the center and a maximal torus of G. The purpose of this paper is to give a direct relation between maximal tori and the center in G and maximal tori in G', as follows.

THEOREM. *Let G be an analytic group and Z the center of G. Let α denote the natural homomorphism $G \to G' = G/Z$. Let H be an analytic subgroup of G containing Z. Then H contains a maximal torus of G if and only if α (H) contains a maximal torus of G'.*

The author would like to thank Soji Kaneyuki for the valuable discussions about the problem in the preparation of this paper.

In § 2 and § 3, we recall results on the automorphism group of G and on maximal tori of G, respectively, for the later use. We divide the proof of Theorem into two parts essentially, Proposition 1 in § 4, and Proposition 2 in § 5, such that Theorem follows from them directly. In § 6, an alternate definition of standard Cartan subgroups will be given as an application.

§ 2. Aut (G)

For an analytic group and for its Lie algebra, we shall use the same capital Roman and capital script letter, respectively. Let G be analytic group, and let Aut (G) denote the group of all bicontinuous automorphisms of G. For the Lie algebra $\mathscr{G}$ of G, let Aut $(\mathscr{G})$ denote the group of all Lie algebra automorphisms of $\mathscr{G}$. Then Aut $(\mathscr{G})$ is an algebraic subgroup in the general linear group $GL(\mathscr{G})$, and for any ρ in Aut (G), there corresponds a unique automorphism $d\rho$ of $\mathscr{G}$ such that