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On oscillation of Volterra integral equations and first order functional differential equations

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1. Introduction

In the functional differential equations with deviating arguments, oscillatory behaviour of solutions plays an important role and has been studied by many authors (cf. [1]). To our knowledge, however, few papers are known in the oscillation theory of integral equations. For a result on the latter problem we refer to Parhi and Misra [4].

Consider the Volterra integral equation

$$x(t) = f(t) - \int_0^t a(t, s)g(s, x(s))\, ds\,, \qquad t \geqq 0\,. \tag{1}$$

In (1), $f: [0, \infty) \to R$ and $g: [0, \infty) \times R \to R$ are continuous, and $a: [0, \infty) \times [0, \infty) \to R$ is such that $a(t, s) = 0$ for $s > t$, $a(t, s) \geqq 0$ for $0 \leqq t < \infty$ and $0 \leqq s \leqq t$. In addition $a(t, s)$ is supposed to be continuous for $0 \leqq t < \infty$ and $0 \leqq s \leqq t$. We consider only the solutions of (1) which exist, are continuous on $[0, \infty)$, and are nontrivial in any neighbourhood of infinity. A solution $x(t)$ of (1) is said to be *oscillatory* if $x(t)$ has zeros for arbitrarily large t; otherwise, a solution $x(t)$ is said to be *nonoscillatory*. This definition is the same as in the case of differential equations.

In this paper we propose some criteria for all solutions of (1) to be oscillatory, which is not considered in [4], and also apply the idea to study the oscillation of functional differential equations of the type

$$x'(t) + \sum_{i=1}^{n} p_i(t)|x(g_i(t))|^\alpha \operatorname{sgn} x(g_i(t)) = q(t)x(t) + r(t)\,, \qquad \alpha > 0\,. \tag{2}$$

We will see that every solution of Volterra integral equation (1) or first order functional differential equation (2) is oscillatory when the strong oscillating forced terms are attached.

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