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On the invariant field of binary octavics

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Introduction

Katsylo and Bogomolov proved in [7, 1, 2] the rationality of the invariant field of binary n-forms over the complex numbers field $\boldsymbol{C}$ for all integers n. The purpose of this paper is to give a set of six generators explicitly along with the classical terminology, i.e. by *symbolic expressions* (cf. [5, 6]), in the case of binary octavics ($n = 8$) (Theorem B). For a general integer n, we show the following:

THEOREM A. *For each even integer* $n \geq 8$ (*see Remark* 1.12 *for odd integers* n), *there is a homogeneous invariant polynomial* M *of degree* 12 *such that the invariant field of binary* n*-forms over* $\boldsymbol{C}$ *is generated by* $n - 2$ (= *transcendence degree over* $\boldsymbol{C}$) *rational functions whose denominators are certain powers of* M.

One of the culminant of the classical invariant theory is [4], in which von Gall shows that the set of 70 covariants (including 9 invariants) listed there, is a complete minimal system of the graded ring of covariants of binary octavics. About twenty years ago, Shioda[8] determined all the syzygy modules of the graded ring of invariants of binary octavics, by means of the symbolic method, generating functions and some technique due to Hilbert. In contrast to the invariant(covariant) rings, there had been, until now, little attempt to calculate algebraically independent generators of the invariant fields of binary forms. The author would like to indicate in the present article that one could apply the classical symbolic method initiated by Gordan and others [5, 6] to express the generators not only of the invariant rings, but also of the invariant fields of binary forms.

It was shown in [7] that the field in question is isomorphic to the invariant field under an action of a subgroup H of $\mathrm{SL}(2, \boldsymbol{C})$. Analizing this isomorphism, we give in §1 a correspondence of H-invariant polynomials to the $\mathrm{SL}(2, \boldsymbol{C})$-invariant rational functions and prove Theorem A. The method used in §1 is a variation of protomorphic functions in the classical invariant theory (Remark 1.13, cf. [3]). In §2 we apply the result of §1 in the case of binary octavics and give a set of generators explicitly. After preparing four lemmas (from Lemma 2.7 to 2.10), the process of constructing the $\mathrm{SL}(2, \boldsymbol{C})$-invariant rational functions