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Radially symmetric solutions of semilinear elliptic equations, existence and Sobolev estimates

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1. Introduction

In this paper we consider radially symmetric solutions to the semilinear elliptic equation

$$\Delta u + f(|x|, u) = 0, \qquad x \in \Omega, \tag{1.1}$$

where $\Omega \equiv \{x \in \boldsymbol{R}^n: |x| < 1\}$, $n \geq 2$, and the function $f(t, u)$ is assumed to be continuous in $[0, 1] \times \boldsymbol{R}$. In order to discuss radially symmetric solutions $u = u(t)$, $t = |x|$, it is natural to convert equation (1.1) to the second order ordinary differential equation

$$u'' + \frac{n-1}{t} u' + f(t, u) = 0, \qquad 0 < t < 1, \tag{1.2}$$

$$u'(0) = 0. \tag{1.3}$$

In the present paper we establish the existence of infinitely many solutions of equation (1.2) under the boundary conditions

$$u'(0) = 0, \qquad au(1) + bu'(1) = 0, \tag{1.4}$$

for any coefficients a and b. Moreover we investigate the Sobolev norms of solutions of the problem (1.2)–(1.3) in conjunction with their zeros. We treat the nonlinear function $f(t, s)$ under superlinear growth conditions and sublinear growth conditions. In the present paper the function f is said to be *superlinear* (in a neighborhood of $s = \pm\infty$) if

$$\lim_{s \to \pm\infty} \frac{f(t, s)}{s} = \infty \qquad \text{uniformly in} \quad t \in [0, 1]. \tag{1.5}$$

On the other hand, f is said to be *sublinear* (in a neighborhood of $s = 0$) if

$$\lim_{s \to 0} \frac{f(t, s)}{s} = \infty \qquad \text{uniformly in} \quad t \in [0, 1]. \tag{1.6}$$