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Derivation of a porous medium equation from many Markovian particles and the propagation of chaos

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§ 0. Introduction

We consider the following nonlinear parabolic equation

$$\frac{\partial u}{\partial t} = \frac{1}{2} \triangle (u^\alpha), \qquad (t > 0,\ x \in \boldsymbol{R}^d), \tag{1}$$

for a given real number $\alpha > 1$, where $\triangle$ is the d-dimensional Laplacian. This equation was introduced by Muskat as an (empirical) equation of the density u of a gas flowing through a homogeneous porous medium and is called a *porous medium equation* ([1]). Analogously to Kac's approach to a Boltzmann equation [10] we introduce a Markov system of many particles as a simple model of the gas. The porous medium equation (1) is derived from the equation for the empirical density of the number of particles. We prove that a macroscopic limit of the empirical density is a solution of (1). We also prove Kac-McKean's propagation of chaos for the system as follows.

Let $S_h = \{(hz_1, \cdots, hz_d): z_1, \cdots, z_d \in \boldsymbol{Z}\}$ be a d-dimensional lattice of the width $h > 0$, and $\tau > 0$ be a unit time. We define a system of N-particles on S_h with the following stochastic interaction. For each integer $n \geq 0$, let

$$X_n^{N,1}, \cdots, X_n^{N,N} \in S_h$$

denote the positions of N-particles at time $n\tau$. If the number of particles at a position $x (\in S_h)$ is $m (\geq 1)$, then each particle at x jumps to one of the nearest neighbor lattice points $x \pm (0, \cdots, 0, \underset{(j)}{h}, 0, \cdots, 0)$ $(j = 1, \cdots, d)$ with probability $\{m/N\}^{\alpha-1}/2d$ and stops on x with probability $1 - \{m/N\}^{\alpha-1}$ independently of the other particles. Thus all N-particles can move at the same time (for detail, see (M. 1), (M. 2) and Remark (3) in § 1).

We consider a macroscopic behaviour of this model. Let $\delta(x, y)$ be Kronecker's δ-function (i.e. $\delta(x, y) = 0$ for $x \neq y$ and $\delta(x, x) = 1$) and define by

$$\bar{X}_n^N(x) = \frac{1}{N} \sum_{i=1}^N \delta(X_n^{N,i}, x), \qquad x \in S_h,$$

the *empirical measure* of the number of particles (on S_h) at time $n\tau$. Suppose