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Dynamics of interfaces in reaction diffusion systems

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1. Introduction

It is observed frequently in nature that a physical system develops in such an inhomogeneous way that in different spatial regions, the system is in distinctive states or it behaves in distinctive manners. Under certain circumstances, these spatial regions may be well-distinguished, and be clearly separated by certain boundaries, which are so-called *interfaces*. Such interfaces form a variety of geometrical patterns, and exhibit significant changes in size, shape and location as time passes.

The interfacial phenomena attract a lot of attention and stimulate continuing activity in natural science. From various points of view, people attempt to understand the underlying mechanism of generation of the interfaces, their internal structure and their dynamical behavior. For example, the classical Stefan problem treats the liquid freezing and the solid melting. The front of shock waves in Riemann problem for fluid flow is another type of interfaces ([16, 26]). Friedrichs [14] presented a fantastic description of many interesting interfacial phenomena arising in physics. More recently, chemists observed rotating spiral waves and expanding target patterns in the well-known Belousov-Zhabotinski reagent ([41, 40]), which leads to extensive mathematical studies of reaction diffusion systems ([10, 36] and references therein). The pigmentation patterns of the shells and the animal coats are also viewed as a kind of interfaces in a theory of biological pattern formation ([27, 30]).

In this paper we are concerned with an interfacial phenomenon in a class of reaction diffusion systems. Mathematically, we study a nonlinear partial differential equation of parabolic type:

$$(1.1a)^{\varepsilon} \qquad \frac{\partial u}{\partial t} = \varepsilon \Delta u + \frac{1}{\varepsilon} f(u, v) \qquad x \in \mathbf{R}^n,\ t > 0,$$

$$(1.1b)^{\varepsilon} \qquad \frac{\partial v}{\partial t} = D \Delta v + g(u, v) \qquad x \in \mathbf{R}^n,\ t > 0,$$

with initial condition

$$(1.2a) \qquad u(x, 0) = \phi(x) \qquad x \in \mathbf{R}^n,$$