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Notes on elements of $U(1, n; C)$

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Introduction

Let C be the field of complex numbers. Let $V = V^{1,n}(C)$ $(n \geqq 1)$ denote the vector space C^{n+1}, together with the unitary structure defined by the Hermitian form

$$\Phi(z^*, w^*) = -\overline{z_0^*}w_0^* + \overline{z_1^*}w_1^* + \cdots + \overline{z_n^*}w_n^*$$

for $z^* = (z_0^*, z_1^*, \ldots, z_n^*)$ and $w^* = (w_0^*, w_1^*, \ldots, w_n^*)$ in V. An automorphism g of V, that is, a linear bijection such that $\Phi(g(z^*), g(w^*)) = \Phi(z^*, w^*)$ for z^*, $w^* \in V$, will be called a *unitary transformation.* We denote the group of all unitary transformations by $U(1, n; C)$. Let $V_0 = \{z^* \in V | \Phi(z^*, z^*) = 0\}$ and $V_- = \{z^* \in V | \Phi(z^*, z^*) < 0\}$. It is clear that V_0 and V_- are invariant under $U(1, n; C)$. Set $V^* = V_- \cup V_0 - \{0\}$. Let $\pi: V^* \to \pi(V^*)$ be the projection map defined by $\pi(z_0^*, z_1^*, \ldots, z_n^*) = (z_1^* z_0^{*-1}, z_2^* z_0^{*-1}, \ldots, z_n^* z_0^{*-1})$. Set $H^n(C) = \pi(V_-)$. Let $\overline{H^n(C)}$ denote the closure of $H^n(C)$ in the projective space $\pi(V^*)$. An element g of $U(1, n; C)$ operates in $\pi(V^*)$, leaving $\overline{H^n(C)}$ invariant. Since $H^n(C)$ is identified with the complex unit ball $B^n = B^n(C) = \{z = (z_1, z_2, \ldots, z_n) \in C^n | \|z\|^2 = \Sigma_{k=1}^n |z_k|^2 < 1\}$, we regard a unitary transformation as a transformation operating on B^n. We introduce the Bergman metric

$$g_{i\bar{j}}(z) = \delta_{ij}(1 - \|z\|^2)^{-1} + \bar{z}_i z_j(1 - \|z\|^2)^{-2}$$

for $z = (z_1, z_2, \ldots, z_n) \in B^n$. Using this metric, we see that the holomorphic sectional curvature is -4. The distance $d(z, w)$ for z, $w \in B^n$ is defined by the use of the Hermitian form Φ as follows:

$$d(z, w) = \cosh^{-1} [|\Phi(z^*, w^*)| \{\Phi(z^*, z^*)\Phi(w^*, w^*)\}^{-1/2}] ,$$

where $z^* \in \pi^{-1}(z)$ and $w^* \in \pi^{-1}(w)$ (see [3; Proposition 2.4.4]).

Many results on Möbius transformations and discrete groups are shown in [1] and [6]. Our purpose of this paper is to find analogous results for elements of $U(1, n; C)$ and discrete subgroups of $U(1, n; C)$. In Section 1 we shall prove that an element of $U(1, n; C)$ can be decomposed into two special

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