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A discrete time interactive exclusive random walk of infinitely many particles on one-dimensional lattices

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§ 1. Introduction and theorems

The aim of this paper is to provide a simple model of discrete time interactive exclusive random walk of infinitely many particles (i.m.p.'s) which yields a simple exclusion process after a simple limiting procedure, and then to show that the method of relative entropy is also applicable to the analysis of stationary measures for a random walk of i.m.p.'s such that i.m.p.'s can move simultaneously.

Suppose $\mathscr{X} \equiv \{0, 1\}^{\mathbf{Z}}$ represents the space of all configurations of indistinguishable i.m.p.'s on one dimensional lattices $\mathbf{Z}$. For a given $\eta \equiv (\cdots \eta_{-1}\eta_0\eta_1 \cdots) \in \mathscr{X}$, the site i is regarded to be occupied by a particle if $\eta_i = 1$. Let $\mathscr{E} = \{\mathrm{e}, \bar{\mathrm{e}}\}^{\mathbf{Z}}$. We associate $\omega \equiv (\cdots \omega_{i-1}\omega_i\omega_{i+1} \cdots) \in \mathscr{E}$ with $\eta \in \mathscr{X}$ and consider that the states η_i and η_{i+1} on the edge $(i, i+1)$ are exchangeable [resp., unexchangeable] if $\omega_i = \mathrm{e}$[resp., $\bar{\mathrm{e}}$]. Then we define an exclusive movement of i.m.p.'s on $\mathbf{Z}$ by the mapping $W_\omega: \mathscr{X} \to \mathscr{X}$ defined by $W_\omega(\eta) = (\cdots \eta'_{-1}\eta'_0\eta'_1 \cdots)$ where

$$\begin{cases} \eta'_i\eta'_{i+1} = \eta_{i+1}\eta_i & \text{iff} \quad \omega_{i-1}\omega_i\omega_{i+1} = \bar{\mathrm{e}}\mathrm{e}\bar{\mathrm{e}}, \\ \eta'_i = \eta_i & \text{otherwise}. \end{cases}$$

More intuitively, the movement of each particle of η is defined through ω of $\mathscr{E}$ in such a way that a particle on the site i moves to the site $i+1$ [resp., $i-1$] if and only if $\omega_{i-1}\omega_i\omega_{i+1} = \bar{\mathrm{e}}\mathrm{e}\bar{\mathrm{e}}$ and $\eta_i = 1$, $\eta_{i+1} = 0$ [resp., $\omega_{i-2}\omega_{i-1}\omega_i = \bar{\mathrm{e}}\mathrm{e}\bar{\mathrm{e}}$ and $\eta_{i-1} = 0$, $\eta_i = 1$]. We remark that if $\eta_i = \eta_{i+1}$, there occurs no change of states on the sites i and $i+1$ even if $\omega_{i-1}\omega_i\omega_{i+1} = \bar{\mathrm{e}}\mathrm{e}\bar{\mathrm{e}}$.

Now suppose that the configuration of i.m.p.'s on $\mathbf{Z}$ at time t is η. Let $\vec{e}(\eta, t)$ be a random element which takes the value in $\mathscr{E}$. Then $W_{\vec{e}(\eta,t)}(\eta)$ defines a random configuration of i.m.p.'s at time $t+1$ which comes from η at time t. In the following we treat the case where the distributions $Q_{(\eta,t)}$ of $\vec{e}(\eta, t)$, $\eta \in \mathscr{X}$, $t = 0, 1, \ldots$, are independent of t, and their common distributions Q_η, $\eta \in \mathscr{X}$, are given as follows: For some fixed constants $0 < \alpha < 1$ and $0 < \beta < 1$