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# Holomorphic functions on the nilpotent subvariety of symmetric spaces

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## Introduction

Let $\mathfrak{g}$ be a complex reductive Lie algebra and let $\mathfrak{g}_{\boldsymbol{R}}$ be a non compact real form of $\mathfrak{g}$. Let $\mathfrak{g}_{\boldsymbol{R}} = \mathfrak{k}_{\boldsymbol{R}} \oplus \mathfrak{p}_{\boldsymbol{R}}$ be a Cartan decomposition of $\mathfrak{g}_{\boldsymbol{R}}$ and let $\mathfrak{g} = \mathfrak{k} \oplus \mathfrak{p}$ be the direct sum obtained by complexifying $\mathfrak{k}_{\boldsymbol{R}}$ and $\mathfrak{p}_{\boldsymbol{R}}$. $G$ denotes the adjoint group of $\mathfrak{g}$ and we put $K_\theta = \{a \in G; \theta a = a\theta\}$, where $\theta: \mathfrak{g} \to \mathfrak{g}$ is a Lie algebra automrophism of order 2 defined by $\theta = 1$ on $\mathfrak{k}$, $\theta = -1$ on $\mathfrak{p}$. $K$ denotes the identity component of $K_\theta$. $S$ denotes the symmetric algebra on $\mathfrak{p}$ and we put $J = \{u \in S; au = u \text{ for any } a \in K_\theta\}$ and $J_+ = \{u \in J; \partial(u)1 = 0\}$. $J'$ denotes the ring of $K$-invariant polynomials and we put $J'_+ = \{f \in J'; f(0) = 0\}$. $\mathcal{O}(\mathfrak{p})$ deotes the space of holomorphic functions on $\mathfrak{p}$. We put $\mathcal{O}_0(\mathfrak{p}) = \{F \in \mathcal{O}(\mathfrak{p}); \partial(u)F = 0 \text{ for any } u \in J_+\}$ and $\mathfrak{N} = \{x \in \mathfrak{p}; h(x) = 0 \text{ for any } h \in J'_+\}$. The space $\mathcal{O}(\mathfrak{N})$ of holomorphic functions on the analytic set $\mathfrak{N}$ (cf. [2]) is equal to $\mathcal{O}(\mathfrak{p})|_{\mathfrak{N}}$ by the Oka-Cartan Theorem.

Consider the restriction mapping $\mathscr{R}: F \to F|_{\mathfrak{N}}$ of $\mathcal{O}_0(\mathfrak{p})$ to $\mathcal{O}(\mathfrak{N})$. In our previous paper [4] we showed that $\mathscr{R}$ is a linear isomrophism of $\mathcal{O}_0(\mathfrak{p})$ onto $\mathcal{O}(\mathfrak{N})$ when $\mathfrak{g} = \mathfrak{so}(d, 1)$ $(d \geqslant 3)$. In this paper we will show that we obtain the same result for any complex reductive Lie algebra.

## 1. Preliminaries.

Let $S'$ be the ring of all polynomial functions on $\mathfrak{p}$ and $S'_n$ be the homogeneous subspace of $S'$ of degree $n$ for $n \in \boldsymbol{Z}_+ = \{0, 1, \cdots\}$. For $f \in S'$ and $a \in K_\theta$, $af \in S'$ is given by $(af)(x) = f(a^{-1}x)$. It is known that any element of $J'$ is invariant under $K_\theta$ (see [1] Proposition 10). It is also known that $J'$ has homogeneous generators $P_1, \cdots, P_r$ such that $P_j|_{\mathfrak{p}_{\boldsymbol{R}}}$ is real valued $(j = 1, \cdots, r)$, where $r = \dim \mathfrak{a}_{\boldsymbol{R}}$ and $\mathfrak{a}_{\boldsymbol{R}}$ is a maximal abelian subalgebra of $\mathfrak{p}_{\boldsymbol{R}}$. $\mathscr{H} = \{f \in S'; \partial(u)f = 0 \text{ for any } u \in J_+\}$ denotes the space of harmonic polynomials on $\mathfrak{p}$. The following lemma is known.

LEMMA 1.1 ([1] *Theorem* 14 *and Lemma* 18). (i) *If* $f \in S'$ *and* $f = 0$ *on* $\mathfrak{N}$, *then* $f \in J'_+S'$, *where* $J'_+S' = \sum_{j=1}^{r} S'P_j$.