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Classification of non-compact real simple generalized Jordan triple systems of the second kind

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Introduction

A non-associative algebra $\mathscr{A}$ satisfying

$$xy = yx \text{ and } x^2(xy) = x(x^2 y) \qquad (x, y \in \mathscr{A})$$

is called a *Jordan algebra*. A triple product (xyz) in $\mathscr{A}$ defined by

$$(xyz) = (xy)z + (zy)x - y(xz)$$

satisfies following two identities:

$$\text{(JTS 1)} \qquad (xyz) = (zyx),$$

$$\text{(JTS 2)} \qquad (uv(xyz)) = ((uvx)yz) - (x(vuy)z) + (xy(uvz)).$$

In general a triple system satisfying these two identities is called a *Jordan triple system*. This definition was given by Meyberg [10], though the word of Jordan triple system had already been used in limited senses [3], [13]. He extended the Koecher's construction of a Lie algebra from a given Jordan algebra to the case of Jordan triple systems. Kantor [6] extended still more this construction to the case of *generalized Jordan triple systems*, which were triple systems satisfying only the identity (JTS 2) by definition. A familiar example of generalized Jordan triple system and not Jordan triple system is the space $M_{m,n}(\boldsymbol{R})$ of $m \times n$ real matrices with the product $(XYZ) = X{}^tYZ$. In a Lie algebra with an involution σ, a subspace U satisfying $[[U, \sigma(U)], U] \subset U$ also becomes a generalized Jordan triple system by the triple product $(xyz) = [[x, \sigma(y)], z]$. Starting from a given generalized Jordan triple system, Kantor constructed a graded Lie algebra, which is called the *Kantor algebra* for the generalized Jordan triple system in this paper. A graded Lie algebra $\mathscr{G} = \sum_{i=-\infty}^{\infty} \mathscr{G}_i$ is said to be *of the n-th kind* ($n > 0$) if $\mathscr{G}_{\pm n} \neq \{0\}$ and $\mathscr{G}_m = \{0\}$ for $|m| > n$. To a Jordan triple system, there associates a graded Lie algebra of the first kind. Since the Lie product in the Kantor algebra was not easy to explain in general style, Yamaguti [14] gave another interpretation for the Kantor algebra in case of the second kind. Moreover he defined a symmetric bilinear form on a generalized Jordan triple system of the second kind. In case of the